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Design of Dual-Rate Controllers using Frequency Domain Techniques

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Resumen

El objetivo de esta contribución es el diseño de un controlador bifrecuencia (DRC) asumiendo el control de un proceso con medidas lentas y actuación rápida, siendo la relación de frecuencias un número entero. El diseño es desarrollado en el dominio frecuencial considerando una técnica original para la obtención de la respuesta en frecuencia de un sistema multifrecuencial. El controlador presentará una estructura no-convencional con dos partes que funcionan a diferentes frecuencias. Se incluyen algunos ejemplos.

Palabras clave: Sistemas invariantes en el tiempo, Sistemas con variación en el tiempo, Diseño analítico, Estructura y limitaciones del controlador, Control de procesos

Design of Dual-Rate Controllers using Frequency

Abstract

The main goal of this contribution is to design a dual-rate controller (DRC) assuming slow measurement and fast control updating with integer relation of frequencies. The design is planned in the frequency domain considering an original technique for dual-rate systems. The DRC is composed by slow and fast parts acting at different frequencies. Detailed examples illustrate the procedure and a comparison with the result from other techniques is also reported.

Keywords: Time-invariant systems, Time-varying system, Analytic design, Controller constraints and structure, Process control

1. Motivation

In digital control systems schemes, sometimes it is difficult to achieve the ideal conditions for sampling rates planned for a desired performance. Physical limitations or economical restrictions prevent to reach the best conditions for the project. In such cases an option is what is known as Multi-Rate Control. There is an important particular case called Dual-Rate (DR) Control in which just two variables are sampling or considered at different frequencies. Moreover it is usual in industrial environments that the process (SISO case) output variable was sampled N times slowly than input variable; chemical plants with slow analyzers performing (J. Zheng and Ge, 2025), visual servoing (Cong, 2023), or networked based control are clear examples of application of these techniques (Salt et al., 2014a). This is what is called the multirate input control (MRIC) problem. In this contribution a new design procedure in the frequency domain is introduced considering a new dual-rate frequency response algorithm introduced by the author (Salt and Sala, 2014). Until now there are different DRC design procedures but time response based on (Salt et al., 2014b). Obviously the DRC $\mathcal{H}^{\infty}$ design methods could be considered like frequency methods, but in that case is difficult to choose a low order controller. Furthermore, it is usual to assume a DR system as a multivariable system (MIMO) with some particularities but the design methods are based on that multivariable conception. In that case the singular value computation from frequency response operator is usually used without phase information.

In this contribution, section 2 points out the basic contents

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for understanding the design process in time and frequency domains. In section 3 the discrete lifting modeling procedure is revisited. The algorithm for computing the DR system frequency response (FR) are also introduced in section 4. Then, section 5 is devoted to review briefly a non-conventional DRC structure and its proper design in time domain. This structure will be also used for frequency domain design and both options will be compared using the FR for DR systems. Section 6 will settle the design procedure and some assumptions to make it easier. Finally a complete example will be described step by step in section 7.

2. Preliminaries

First of all some basic multirate definitions, operations and properties are needed. F^T will denote either the Z-transform of the sequence $\{f(kT)\}$ obtained by sampling the continuous signal $f(t)$ or the sampling rate transformation of a discrete signal F. More explanation will be given below.

$$F^T(z) = \mathcal{Z}^T[\{f(kT)\}] = \sum_{k=0}^{\infty} f(kT)z^{-k} \tag{1}$$

in the same way, if the sampling period is NT

$$F^{NT}(z^N) = \mathcal{Z}^{NT}[\{f(kT)\}] = \sum_{k=0}^{\infty} f(kNT)z^{-kN} \tag{2}$$

Now it is defined the upsampling (from NT to T) and downsampling (from T to NT) transforms:

$$[F^{NT}(z^N)]^T = \sum_{k=0}^{\infty} \bar{f}(kT)z^{-kN} = \begin{cases} \bar{f}(kT) = f(kT); & \forall k = \lambda N \\ \bar{f}(kT) = 0; & \forall k \neq \lambda N \end{cases} \tag{3}$$

$$[F^T(z)]^{NT} = \hat{Y}^{NT}(z^N) = \sum_{k=0}^{\infty} f(kNT)z^{-kN} = F^{NT}(z^N) \tag{4}$$

Some upsampling-downsamling properties are (Salt and Albertos, 2005):

$$\begin{aligned} [X^T Y^T]^{NT} &\neq [X^T]^{NT}[Y^T]^{NT} \\ [X^{NT} Y^{NT}]^T &= [X^{NT}]^T[Y^{NT}]^T \\ [X^T[Y^{NT}]^T]^{NT} &= [X^T]^{NT} Y^{NT} \end{aligned} \tag{5}$$

3. Discrete Lifting Revisited

A well-know method for modeling multirate systems is the so called Lifting. From the Kranc's idea (Vector Switch Decomposition) (Kranc, 1957), lifting is based on lifted the signals in one frame. That frame is the period at which every sampling sequence is repeated (metaperiod). In general, for a DR system with input and output sampling periods T_u and T_y (rationally related) respectively, $T_0 = lcm(T_u, T_y)$ will be the metaperiod and the input and output signals will be lifted in vectors with size N_u and N_y respectively, such that $T_0 = T_u N_u = T_y N_y$. In the case of this contribution, schemes with T and NT samplers will be considered. Therefore, the metaperiod will be $T_0 = NT$ and the discrete T signals will be lifted in size N vectors. It is usual to model the behaviour of the DR system characterized via a "lifted"transfer function matrix:

$$y_l(z^N) = \tilde{G}(z^N)u_l(z^N) \tag{6}$$

where the subindex «l» denotes «lifted» and z^N is referred to the z variable at least common period T_0 as was described in the previous paragraph. It is possible to assume internal or external representations for lifting models; in (Francis and Georgiou, 1988) the links between them are described. In this section in order to expose the DR FR algorithm, the internal representation will be the proper one. In order to explain how is deduced, a general strictly proper continuous system preceded by an ZOH and samplings period T_u for input and T_y for output will be considered. It is possible to obtain the T_0 lifting representation from a T ZOH-discretization of the process being $T = gcd(T_u, T_y)$. If that discretization is $(A, B, C, 0)$, the lifting model can be deduced by repeated evaluations of the equations at sampling period T. The result will be:

$$\begin{aligned} y(kT_0 + \zeta T) =& Cx(kT_0 + \zeta T) = \\ &= C[A^{\zeta} x(kT_0) + A^{\zeta-1} Bu(kT_0) + \\ &+ A^{\zeta-2} Bu(kT_0 + T) + \ldots + Bu(kT_0 + (\zeta-1)T)] \end{aligned} \tag{7}$$

for $\zeta = 1, \ldots, (N_u - 1)N_y$,. However, the zero-order-hold entails

$$\begin{aligned} &u(kT_0 + dN_yT) = u(kT_0 + (dN_y + 1)T) = \ldots \\ &\cdots = u[kT_0 + ((d+1)N_y - 1)T] \\ &\forall d = 0, 1 \ldots, (N_u - 1) \end{aligned} \tag{8}$$

The lifted matrices (A_l, B_l, C_l, D_l) are obtained by suitably stacking the results from the above equations.

4. Dual-Rate Frequency Response

When a DRS is studied in the frequency domain, it is usual to consider it like a multivariable system (see lifting modeling). A classical perspective is to analyze the singular value decomposition of the RF of lifted matrix. The problem with such a procedure is that some information is lost (it is not accesible the phase information). In (Salt and Sala, 2014) an efficient algorithm for computing the DR system FR was introduced. The result obtained in that contribution establishes that the output $y(k)$, when $u(k) = e^{j\omega T_u k}$, of a SISO dual-rate ($N_u T_u = N_y T_y$) lifted system $y_l(z) = G_{lifted}(z)u_l(z)$ is a collection of components $y_r(k) = \bar{y}_r e^{jT_y\omega_r k}$ of frequencies $\omega_r = \omega + 2\omega_y^s r/N_y$, for $r = 0, \ldots, (N_y - 1)$, with $\omega_y^s = \pi/T_y$, and $\bar{y}_r$ is given by:

$$\bar{y}_r = \frac{1}{N_y} \sum_{p=0}^{N_y-1} \sum_{q=0}^{N_u-1} G_{pq}(e^{j\omega_r T_y N_y}) e^{-jT_y\omega_r p} e^{j\omega T_u q} \tag{9}$$

It is possible to check that, from (9), the components will be given by the product of the frequency response of a left factor:

$$[1 \; z^{-1} \; z^{-2} \; \ldots \; z^{-(N_y-1)}]G_{lifted}(z^{N_y}) \tag{10}$$

replacing $z = e^{j\omega_r T_y}$, which gives a row vector, and the right factor (column vector)

$$(1 \; z \; z^2 \; \ldots z^{N_u-1})^T \tag{11}$$

replacing $z = e^{j\omega T_u}$.

If it was stated before, $T_u = T$ and $T_y = T_0 = NT$, there will be N_y components which can be read from only one Bode plot from the lifted matrix [1]. For an input frequency d Rad/s, the readings must be done at $d, d + w_s, \dots d + (N-1)w_s$ being $w_s = \frac{2\pi}{NT}$. This result will be assumed to detect the existence of ripple applying interlacing implementation, but as it is easy to understand does not lead to pure frequency response, because the "N sinusoids addition is not a sinusoid. For that purposes the metaperiod sum must be considered:

$$\begin{aligned} y(kT_y) =& A_1 sin(wkT_y + \varphi_1) + A_2 sin((w + N_u w_s)kT_y + \varphi_2) + \dots + \\ &+ A_{N_y} sin((w + (N_y - 1)N_u w_s)kT_y + \varphi_{N_y}) \end{aligned} \tag{12}$$

As it was said this is not a pure sine signal. However if the decomposition of each component is observed:

$$\begin{aligned} A_v sin(w_v kT_y + \varphi_v) =& A_v sin(w_v kT_0 + \varphi_v) + \\ & z_{T_y}^{-1} A_v sin(w_v kT_0 + (w_v T_y + \varphi_v)) + \dots \\ & z_{T_y}^{-(N_y-1)} A_v sin(w_v kT_0 + (w_v(N_y - 1)T_y + \varphi_v)) \\ & for \quad v = 1, \dots, (N_y - 1) \end{aligned} \tag{13}$$

So, adding the contributions of each component at kT_0, that is the downsampling of $y(kT_y)$, the T_0 approximation is obtained: T

$$\begin{aligned} y^{T_0}(kT_y) =& A_1 sin(wkT_0 + \varphi_1) + \\ &+ A_2 sin(wkT_0 + (w_s T_0 + \varphi_v)) + \dots \\ &+ A_{N_y} sin((wkT_0 + ((N_y - 1)w_s T_0 + \varphi_{N_y})) \end{aligned} \tag{14}$$

that is a pure sinusoid. In this process, the detail is lost but allows to consider the classical frequency representation. However, in this paper, either the components and the T_0 sum will be assumed. The components lead to a suitable mean to identify ripple in the blocks contributions; actually this is a common problem in multirate systems if the sampling periods are not appropriate.

5. Non-conventional structure Dual-Rate Controller

Some works from author et al introduced a special structure for a DRC scheme in a MRIC problem. The detailed explanation can be read at (Salt and Albertos, 2005; Salt et al., 2014b). In (Salt and Albertos, 2005) is exposed the time-design based procedure. Basically, this DRC is composed by a slow part (working at the low frequency or measurement frequency) G_1^{NT}, an expansion and digital holder $H^{NT,T}$ to transform the slow signal output from slow DRC part, and a fast part (performing at updating control frequency or high frequency) G_2^T. In that contribution a model based design was introduced. The objective was to achieve a behaviour settled by continuous closed loop transfer function $M(s)$. In addition the ideal closed loop response follows the $M(s)$ ZOH discretization at fast and slow frequencies. Some problems with ripple could occur but there are ways for avoiding as is pointed out in the cited contribution. The non conventional DRC parts, using the notation introduced in section 2 are :

$$G_1^{NT}(z^N) = \frac{1}{1 - M^{NT}(z^N)} \tag{15}$$

$$G_2^T = M^T(z)/G_p^T(z) \tag{16}$$

$$H^{NT,T}(z) = \frac{R^T(z)}{(R^{NT})^T(z)} = \frac{1 - z^{-N}}{1 - z^{-1}} \tag{17}$$

In Figure 1 is depicted the configuration. Note that the algebra introduced in section 2 was considered.

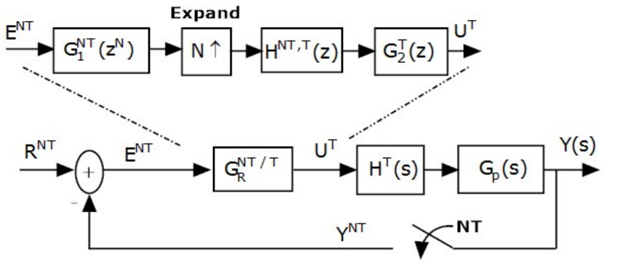

Figura 1: Closed Loop Dual-Rate Control. DR Controller Structure

There are different ways to design this DR controller (Salt et al., 2014b; Salt and Albertos, 2005).

Now it will be tried to design both parts considering a frequency domain procedure. Nowadays, the computation methods allow to use them for a direct digital control design without some approximation rules (Franklin et al., 1998). One of the advantages of the proposed procedure is to obtain parts with low order establishing some specifications to reach. The time-based method leads to high order controller' parts.

5.1. Example

Now, it is introduced an example to show the application of every concept exposed until this point in the current contribution. Specifically a plant:

$$G(s) = \frac{1{,}5}{(s + 0{,}5)(s + 1{,}5)} \tag{18}$$

is assumed. The goal is to reach the closed loop transfer function $M(s)$ obtained by means of a PID controller.

$$M(s) = \frac{2{,}4(s + 4{,}66)(s + 0{,}335)}{(s + 0{,}33)(s^2 + 4s + 11{,}4)} \tag{19}$$

The frame period is $T_0 = 0{,}3$ and a value of $N = 3$ is considered, that is, the fast control is updated at $T = 0{,}1$. When the non-conventional structure DRC is designed, the following results are reached:

$$\begin{aligned} G_1^{NT}(z^N) &= \frac{z^3 - 1{,}656z^2 + 0{,}0{,}9741z - 0{,}2671}{z^3 - 2{,}328z^2 + 1{,}72z - 0{,}3913} \\ G_2^T(z) &= \frac{23{,}55z^4 - 74{,}51z^3 + 86{,}81z^2 - 43{,}92z + 8{,}08}{z^4 - 1{,}402z^3 - 0{,}2546z^2 + 1{,}141z - 0{,}4677} \end{aligned} \tag{20}$$

In Figure 2 is depicted the output when a step in the closed loop is applied. The closed loop DRS Bode diagram assuming

[1] It is possible to obtain the frequency response either in internal or the equivalent external representation

$3T$ was drawn by means of the explained technique leads to figure 3. As it can be seen there are peaks at $w = 20{,}94R/s$ that can not be explained clearly in the zone $\frac{\pi}{T_0} = 10{,}47R/s$. Obviously, the FR matrix maximum singular value matches exactly the outputs sum (that we called phasor). The phase diagram is only known using the explained procedure. So, the components are necessary to explain ripple in the closed loop response. In order to understand the results, an additional comment must be done. The smoothness of the Bode gain function of a component gives a ripple-free response. There are other two cases where this function is not smooth, giving an oscillating response. This problem appears if the gain Bode has a sharp peak or valley in the folding frequency. In the case of the peak that frequency component of the input signal is strongly amplified as it is easy to understand. The valley in the gain function (gain nearly zero at the folding frequency) means that the frequency component of the input signal at $w_s/2$ is eliminated producing an oscillation in the output of a maximum amplitude equal to the content in that frequency of the input.

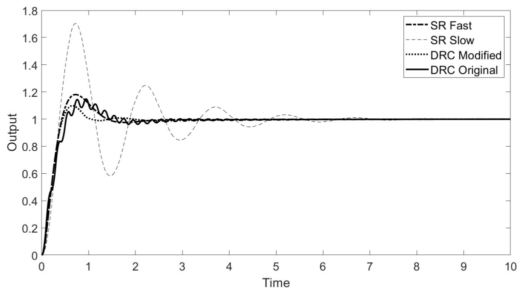

Figura 2: Closed Loop Dual-Rate Control with SR and DR Controllers

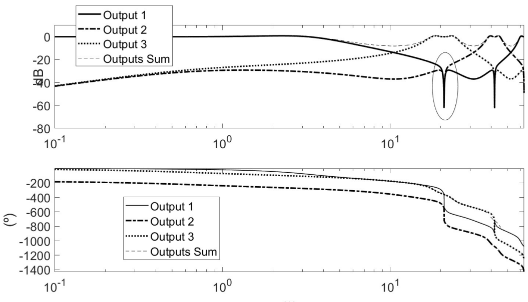

Figura 3: Closed Loop Dual-Rate Bode diagrams

6. Frequency Domain Design

First of all it must be clarified the design objective. Usually when a DRC is projected is because a single rate control was planned but different problems make unfeasible to apply those ideal sampling frequency conditions. Usually the main restriction is the output variable measurement frequency. One solution is to increase both input-output frequencies, but this alternative could easily results in losing performance or even instability. Therefore the DR option can be considered. Logically the objective will be to obtain the same original projected performance, that is the fast single rate control conditions. As such, the main goal will be to design a non conventional structure DRC for reaching the same frequency fast ideal conditions.

The DRC contains:

- a single rate fast part and consequently easy to design following classical procedures
- a slow-fast part with slow part and an essential rate converters that will avoid ripples due to expansion step.

In order to consider the fast part isolated in the design procedure, it will be necessary to design the slow part containing the known rate converter. In this case the FR DR system algorithm must be used because a slow input a fast output is considered. As it was explained, N components will appear but as it will be justified, the first component is decisive at low frequencies because the magnitude contribution of the rest of components is negligible in that zone. Due to this approximation a post analysis step will be mandatory. The proposed design procedure is quite similar to a classical one. The development is shown in the next section by means of an example.

7. Design Example

Given the process with transfer function (18), a continuous PID control was designed in order to achieve some specifications.

$$G_{Rc}(s) = 7{,}5\left(1 + 0{,}2s + \frac{1}{3s}\right) \tag{21}$$

An ideal sampling period for the control project would be $T = 0{,}2$. Using the discretization proposed by Iserman (1990):

$$G_{Rd}(z) = \frac{q_0 z^2 + q_1 z + q_2}{z(z-1)} \tag{22}$$

with $q_0 = 15{,}46$, $q_1 = -22{,}96$, $q_2 = 8{,}$. In these conditions, the discrete Bode diagrams show the magnitude and phase margins that will be the goal of the frequency domain DRC design. In this case the gain margin is $8{,}43dB$ at frequency $7{,}55R/s$ and the phase margin is $35°$ at $3{,}77R/s$.

For the DRC the different parts orders will be previously fixed in order to consider a low order DRC. The classical PID discretization has been chosen:

$$G_{PI}(z) = K\frac{z-a}{z-1} \qquad G_{PD}(z) = K\frac{z-b}{z} \tag{23}$$

that is, the design of two parameters (real numbers) will be necessary. It must be clear that the gain K will be assigned to either integral or derivative parts; practical situations make useful to assign it to the integral parts although the total performance is identical whatever the option selected. Some problems prevent to use this rate for measurement frequency. A $N = 2$, that is, a sampling period of $NT = 0{,}4$ is assumed. The Bode diagram in Figure 4 shows a deep drop of the stability margins next to instability. Therefore the DR option is activated.

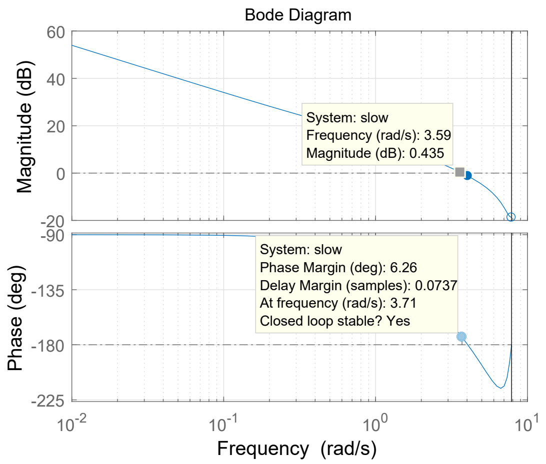

Figura 4: Open Loop Slow Single Rate Control

First, the DR open loop Bode diagram is drawn in Figure 5 using the introduced algorithm with output inter-sampling $T/3$ and considering an unit transfer functions for slow and fast parts. This step allows to know how far is the desired performance from the basic DR loop.

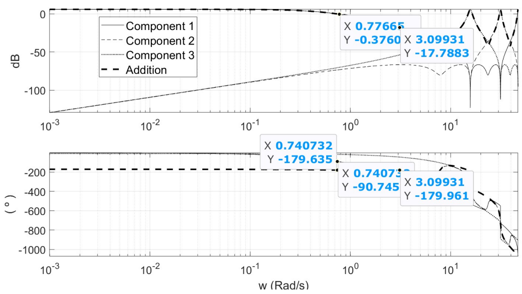

Figura 5: Open Loop Dual-Rate Control with unit DRC

As it is shown in Figure 5, the first component is clearly the dominant magnitude in a large frequency range but it is not in phase contribution with stability margins very different than the desired ones. The system is obviously type 0.

It can be followed different procedures for obtaining a PID type dual-rate controller. First of all, it must be said that frequency response design of discrete systems was usually faced using a transformation from $\mathcal{Z}$ plane to $\mathcal{W}$ plane and using classical continuous time frequency response design steps. However current software CAD control systems design packages allows to carry out the design directly in the $\mathcal{Z}$ plane. This will be the way of this contribution. There are different ideas to perform this direct discrete PID type design.

The first step will be to design the set composed by slow part DRC and the rate converter (assuming a step input reference). In this case it has been plotted the first component DR Bode for different values of parameter $a \in [0{,}5, 0{,}9]$, that is the zero of the integral part. First $K = 1$ is considered.

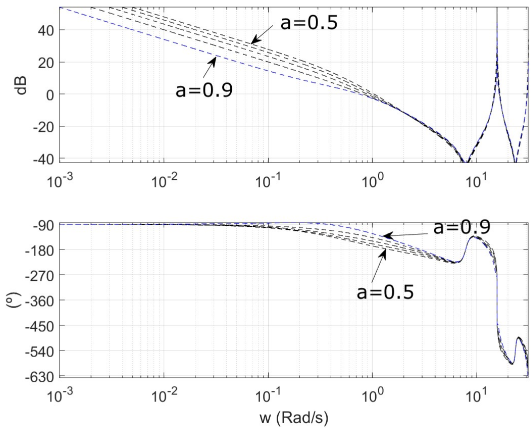

Figura 6: Open Loop Dual-Rate Control with unit PD and different I

As it can be seen in Figure 6 the phase has greater values for $a = 0{,}9$ and magnitude values are practically the same in the derivative design frequency range. In this figure, the phasor (slow frequency) has been depicted.

In figure 6 is plotted the phasor Bode diagrams for DR open loop including a unit fast controller and slow rate PI controller. Obviously, the fast PD block must compensate the phase difference form the desired value (35°). In figure 7 the phasor open loop Bode diagrams have been depicted considering PD with different $b \in [0{,}5, 0{,}9]$. As it can be seen for $3{,}77 R/s$, it is obtained a phase of $-141{,}1°$ for $b = 0{,}8$ and $-149{,}4°$ for $b = 0{,}7$. The desired phase is approximately reached at $b = 0{,}76$, but for this frequency, the magnitude is of $-12{,}2dB$. So, an increase of $10^{\frac{12{,}2}{20}} = 4{,}07$ should be assumed. In figure 8 is shown the gain adjustment. As it is read in the same Figure, the gain margin is $6{,}512dB$. If a greater gain margin is desired, a new gain adjustment should be assumed, but the $0dB$ crossover frequency will be moved to a lower value. Specifically if the same gain margin that the fast control is the target value $8{,}43dB$, a decrease of $1{,}918dB$, that is of about 0,8.

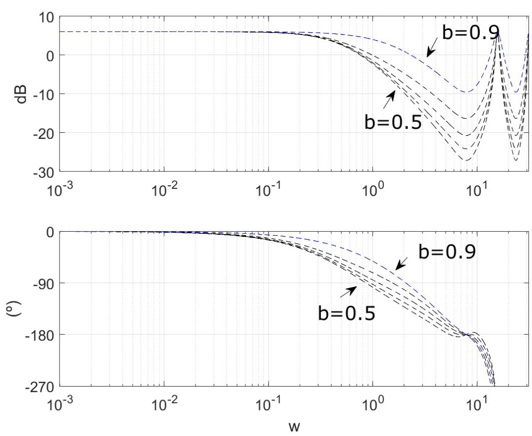

Figura 7: Open Loop Dual-Rate Control with I ($a = 0{,}9$) and different PD

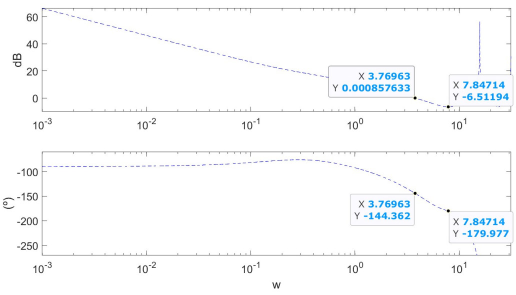

Figura 8: Open Loop Dual-Rate Control with I ($a = 0{,}9$) and PD $b = 0{,}76$ with gain adjustment

Note that, initially a different I parameter (a) could be fixed implying a different PD parameters (b and gain) values. If the process was repeated considering $a = 0{,}7$, $b = 0{,}85$ should be obtained, with gain margin of $6{,}129dB$ and therefore with a gain adjustment of 0,7673. Figure 9 shows the step response of the system for all these cases. As it is deduced, a lower value of I parameter improves the output. The DR responses are a good approximation with a significant decrease of the order of the controllers. In fact, the DRC designed taking into account the procedure introduced in section 5, leads to:

$$
\begin{aligned}
G_1^{NT}(z^N) &= \frac{z^3 - 1{,}296z^2 + 0{,}5635z - 0{,}172}{z^3 - 2{,}131z^2 + 1{,}365z - 0{,}2344} \\
G_2^{T}(z) &= \frac{17{,}95z^4 - 53{,}04z^3 + 57{,}01z^2 - 26{,}11z + 4{,}216}{z^4 - 1{,}203z^3 - 0{,}3607z^2 + 0{,}909z - 0{,}363}
\end{aligned} \tag{24}
$$

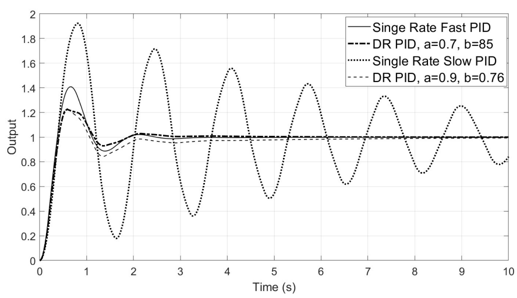

Figura 9: Closed Loop Time Response

In Figure 10 is depicted the comparison among fast single rate controller and both DRC designed either with frequency design techniques or model based from original continuous PID controller. As it can be seen, the model based DRC for this case $N = 2$ induces ripple in closed loop response like it is clear analyzing the DR Bode diagrams shown in figure 11. The magnitude has a sharp valley at $\frac{\pi}{T} = 31{,}416R/s$ that denotes this phenomenon as it was explained in the example of section 5. This detection is difficult by observation of the sum of components curve.

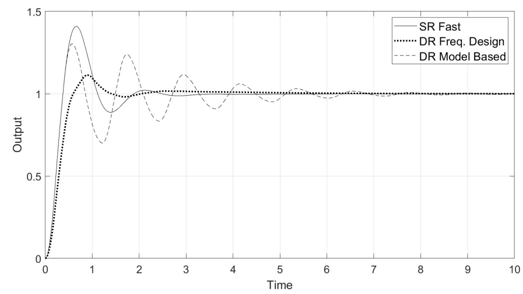

Figura 10: Closed Loop Time Response. DRC comparison

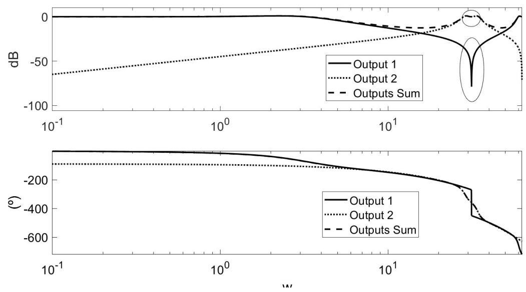

Figura 11: Closed Loop Bode Diagrams $N = 2$

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