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Article

DEA model for optimising economic policies of European Union countries

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Abstract. This study focuses on the problem of optimal choice between unemployment and inflation in view of the international position of the country. The proposed DEA model was tested on the example of statistics of European Union countries for 2023. This year, the efficient frontier of countries' states (the international Phillips DEA line) was formed by the Czechia, Germany, the Netherlands and Belgium. National Phillips DEA lines were built by parallel moving of the international frontier. Optimal states were determined by minimising the quadratic social loss function. A point with minimum indicator levels (the "ideal" state) was the centre of social loss curves. Deviations of actual states from the "ideal" one were normalised by dividing by the difference in coordinates of this state and the binomial mean point of efficient countries. According to the proposed optimisation efficiency coefficient, the best strategy for Bulgaria, Ireland, Malta and Slovenia would be to move directly to the state of the international optimum. Germany and the Netherlands would not benefit from optimising their state. The proposed model can be used in countries' economic policies and their ranking.

Keywords: international comparative studies; DEA model; Phillips curve; social loss function; normalisation methods
JEL classification: F620; E310; C540

1. Introduction

Since the Great Depression of 1929-1933, inflation and unemployment remain the two key problems of modern national economies. Both of them have not only purely economic, but also great social significance. Their combination largely determines the degree of satisfaction (or dissatisfaction) of the country's population with the policies of its authorities. In democratic countries, the combination of "inflation - unemployment" is a significant factor in the electoral process. In authoritarian and totalitarian countries, the aggravation of these problems often led to the radicalisation of public sentiment and the violent change of the outdated political regime.

In the context of globalisation of the world economy, national levels of unemployment and inflation are no longer perceived only as an internal affair of individual countries. These indicators are taken into account in one way or another by the population, business and authorities of other countries - neighbours, partners or competitors. The inflation rate is used in many financial and economic calculations, in particular in determining the real interest rate. The difference in national real interest rates is one of the main factors of legal (and sometimes illegal) international movement of capital. Similarly, the differentiation of national levels of employment and social security significantly affects the international migration of the population, which has recently become an acute global problem. In some European countries, the policy of encouraging the influx of foreign low-skilled population instead of accelerating economic development has led to increased unemployment and ethno-social tension.

Short and long run Phillips curves are a classic tool for analysing the relationship between inflation and unemployment. The long run curve is determined by the natural unemployment level - the average level of unemployment over a long period of time. It is believed that this level of unemployment does not change the inflation rate and therefore can be used as a basic indicator in the policy of the country's government and its central bank. Over historical time intervals, the natural level of unemployment can change - under the influence of both internal and external factors. However, in a globalised world economy, the question arises as to how "natural" the national natural level is. The same applies to the target inflation rate indicator. In view of this, the problem of constructing international analogues of the short run national Phillips curve and determining optimal states of individual countries on them, in which a minimum of a certain international social loss function is achieved, is becoming relevant.

The rest of the paper consists of the following sections. Section 2 presents a literature review. Section 3 contains data specifications and methodology. Empirical results and preliminary analyses are presented in Section 4. Section 5 is devoted to a discussion of research results for individual EU countries. The last section, Conclusions, contains a brief presentation of the applied methodology, research findings and practical value of the proposed model.

The novelty of the proposed study is that it introduces and combines for the first time in one DEA model: a) the international efficiency frontier "inflation - unemployment" (the Phillips' DEA line) and similar national lines; b) the quadratic function of international social losses, normalised by the DEA method; c) DEA optimisation efficiency coefficients, which are the criteria for choosing the international strategy of a certain country regarding the trade-off between inflation and unemployment.

2. Literature review

The social loss function is a key component of optimal monetary policy models. Because of this, it appears in many works as a function of central bank losses. Usually certain indicators of inflation π_t, gross domestic product y_t, unemployment u_t, real or nominal interest rates - r_t, i_t are arguments of this function. Linear-quadratic functions are one of the most widely used loss functions. A study of the history of quadratic loss functions and monetary policy rules has shown that eras of their application provided greater economic stability (Teryoshin, 2023).

In the simplest case, the loss function has two arguments. Thus, Aikman et al. (2023) use the function

$$\mathbb{L}_t(y_t;\pi_t)=\frac{1}{2}\left[(\pi_t-\pi^*)^2+\lambda_y(y_t-y^*)^2\right] \qquad [1]$$

where π^*, y^* are target levels of inflation and output, and λ_y is the weighting coefficient.

In a particular case, zero inflation rate and potential output can be chosen as target values:

$$\mathbb{L}_t(y_t;\pi_t)=\pi_t^2+\lambda_y(\hat{y}_t^{GAP})^2 \qquad [2]$$

where y^{GAP} is the output gap. The equivalent stochastic equation (Benchimol, 2024) has the form

$$\mathbb{L}_t=\mathrm{var}(\widehat{\pi}_t)+\lambda_y\mathrm{var}(\hat{y}_t^{GAP}) \qquad [3]$$

where $\mathrm{var}(\cdot)$ is the variance operator.

According to Chadha & Schellekens (1999) classification, quadratic losses reflect increasing absolute risk aversion

$$\mathbb{L}(\pi_{t+1}-\pi^*)=\frac{1}{2}(\pi_{t+1}-\pi^*)^2 \qquad [4]$$

where π^* is the socially optimal rate of inflation. The infinite sum of such discounted losses forms an intertemporal function

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\delta^{t+1}\,\mathbb{L}(\pi_{t+1}-\pi^*)\right\}\longrightarrow\min \qquad [5]$$

where $\mathbb{E}$ is the expectation operator and δ is the discount factor.

Djeutem et al. (2022) use the function of constant absolute risk aversion (CARA) to account for the household's choice between labour and consumption.

Kiley (2024) conducts a comparative analysis of three quadratic loss functions in view of ELB (Effective Lower Bound) risks and the need for a strong labour market. In all three functions, the first term takes into account inflation deviations from the target level. In the first function, the second term reflects the deviation from potential output, and in the second one, the deviation from the socially efficient level of output. In the third function, only the drop in output below potential one is considered as losses:

$$\mathbb{L}_t=(\pi_t-\pi^*)^2+\lambda_y[\min(y_t^{GAP};0)]^2 \qquad [6]$$

The model used by the Swedish Central Bank considers a set of scenarios with different interest rates (Svensson, 2009; Ingves, 2011). For each scenario, mean squares of forecast gaps of inflation and output are calculated. On the plane of conditional variances, the lower left boundary of the set of scenarios forms a “front”. The optimal scenario will be the point of tangency of this “front” with the intertemporal forecast loss line:

$$\mathbb{L}_t=\sum_{\tau=0}^{\infty}\delta^{\tau}\left(\overline{\pi}_{t+\tau,t}-\pi^*\right)^2+\lambda_y\sum_{\tau=0}^{\infty}\delta^{\tau}\left(\overline{y}_{t+\tau,t}-\overline{y}_{t+\tau,t}^{\mathrm{pot}}\right)^2, \qquad 0<\delta\le 1 \qquad [7]$$

where δ is the discount factor.

Gross & Leigh (2022) consider a three-factor loss function

$$\mathbb{L}_t=\lambda_\pi(\pi_t-\pi^*)^2+\lambda_u(u_t-u^*)^2+\lambda_{ncr}(ncr_t-ncr_{t-1})^2 \qquad [8]$$

where ncr is the nominal cash rate.

Yuan & Miller (2009) show that in the New-Keynesian Model with an elementary loss function and cost-push shocks, systematic evolution paths converge to long run target indicators. Hinterlang & Tänzer (2021) use a two-factor quadratic function in a neural network model of the economy.

At the same time, a number of authors point out certain problems caused by the use of a quadratic function. In particular, monetary expansion under conditions of underemployment creates a negative correlation between inflation and the output gap, which masks the Phillips curve (McLeay & Tenreyro, 2019). As Dorich et al. (2021) note, in Canada, the value of $\pi^* = 2\%$ remains unchanged since 1995. Despite its stabilising effect, a low target inflation rate also has significant negative consequences - stimulation of excessive risk and debt accumulation. To solve these problems, the Bank of Canada periodically changes the set of target indicators.

In the model of Barnichon & Mesters (2023), inflation gap and unemployment rate gap are the arguments of the intertemporal loss function. According to Bonciani & Oh (2025), targeting of solely inflation and output excessively expands the central bank's balance sheet. To avoid this drawback, the authors propose to include a third term in the loss function, which reflects the inequality of consumption of savers and borrowers.

In the model of Deák et al. (2024), optimal monetary policy is presented as the result of a game between a leader government and its follower - an independent central bank. The quadratic loss function contains the deviations of trend inflation, nominal gross interest rate, and economic growth. The central bank should maximise the actual intertemporal welfare of the household that is subject to the ZLB (zero-lower-bound) constraint.

However, as Dziuda & Pflueger (2021) prove, consideration of the electability of democratic governments contradicts the role of a central bank as a social planner. According to the authors' two-period model, an apolitical central bank minimises the quadratic loss function. In this function, the socially optimal inflation rate is equal to zero, and the socially optimal unemployment rate is negative:

$$\mathbb{L}_t = \frac{1}{2}[(u_t - u^*)^2 + \lambda_\pi \pi_t^2], \quad \pi^* = 0\,, \quad u^* < 0 \qquad [9]$$

Fully rational voters evaluate incumbent politicians by the unemployment criterion, and governments prefer more inflation-averse central banks. As a result of inflation targeting, economic situation worsens, which contributes to the re-election of lower-quality politicians.

Quadratic intertemporal loss functions are also used in international economic research. For example, Caldara et al. (2024) show that synchronous monetary tightening has a greater impact on output than on inflation. Optimal targeting rules under international cooperation are the subject of research by Corsetti et al. (2023). Various scenarios of international impacts are analysed in Keynesian-type models. For example, in the model of Chen et al. (2023), the spread of shocks in the global economy leads to interdependence of national optimal targeting rules. Ida & Iiboshi (2021) analyse, in particular, the situation when households are risk-averse and central banks seek to minimise national, rather than global, losses.

Some authors question the quadratic form of the loss function itself. For example, Haavio et al. (2024) present evidence of the actual asymmetry of the European Central Bank function in 1999-2021. Karadi et al. (2024) consider optimal monetary policy during large cost-push shocks and total factor productivity shocks. In their model, the Phillips curve is nonlinear, and weighting coefficients are variable. Mayer (2003) notes that in the long run, excess target output and lower (compared to target one) inflation should be considered not as losses, but as gains. Mazelis et al. (2023) conclude that central bank operations are unable to completely neutralise the destabilising effect of the ELB, provided that the volume of asset purchases is acceptable. Sarlin & von Schweinitz (2021) propose an ex-ante optimisation model that better prevents recurring financial instabilities. In the model of Pfajfar & Winkler (2024), the loss function is a power one, and its exponent η is calculated based on a sociological survey:

$$\mathbb{L}_i(\pi, u) = (\pi - \pi_i^*)^\eta + \lambda_u (u - u_i^*)^\eta \quad [10]$$

where i is a respondent's number.

Given the nature of the proposed study, works, in which the international position of countries is investigated using geometric methods, are of considerable interest. For example, Prachowny (1990) maps the states of 14 OECD countries on the "u - π" plane for the period 1973-1987. A study of European Monetary Union (EMU) countries by Florio et al. (2024) confirms a theoretical conclusion that globalisation reduces the slope of the Phillips curve. Eser et al. (2020) determine the slope of the structural Phillips curve for euro area countries, using, in particular, cross-country variation.

Pham & Sala (2022) apply the Granger-causality network graph to analyse inflation and unemployment in G7 countries and in Spain. Itskhoki & Mukhin (2023) visualise their own model of optimal policy on the output gap and exchange rate volatility plane. On this plane, the centre of elliptic social loss curves is located at the borderline point of the classic trilemma constraint for an open economy. Sakr et al. (2024) use seven DEA (Data Envelopment Analysis) models and the Malmquist productivity index to measure countries' relative efficiencies in using foreign aid from 2002 to 2020. Hoon & Ho (2007) investigate how much of the big movements in the national unemployment rate can be explained by non-monetary factors, primarily by the distance to World Technology Frontier (WTF).

The vast majority of modern monetary policy models are based on a parametric approach to constructing the functions used. First of all, this concerns the determination of the shape of the short run Phillips curve. However, as Buchmann (2009) shows in a study of euro area countries, less stringent assumptions on the correspondence between inflation, expectations, and marginal costs are quite justified. The author derives expectations from the European Commission's Consumer Survey data. The obtained results reveal a half-year lag of the impact of inflation expectations and a significant nonlinearity of the short run curve, which casts doubt on the validity of its New-Keynesian form.

The approach of McKay & Wolf (2023) is based on the quadratic loss function and does not require following a particular parametric model. The optimal strategies thus obtained are robust to the Lucas critique.

Given the wide variety of existing models and the difficulty of determining the best of them, the European Central Bank (ECB, 2021) considers it appropriate to combine known models, unobserved data and other information to obtain averaged forecast. This feature is pointed out by Cancelo et al. (2011). The authors show that models with different Taylor rules in different countries better explain the ECB's monetary policy.

3. Data and methodology

3.1. Data specifications

Traditionally, the coordinate system "unemployment rate (u) - inflation rate (π)" is used to construct national Phillips curves. The proposed study uses new coordinate systems, in which both indicators have the same algebraic form. In the first coordinate system, inflation is characterised by the logarithm of the price index P, and unemployment - by the logarithm of the ratio of a labour force L to a number of employees E, which can be interpreted as the logarithm of the maximum possible employment index:

$$x_{log} = \ln\left(\frac{\max E_t}{E_t}\right) = \ln\left(\frac{L_t}{L_t - U_t}\right) \quad [11]$$

$$y_{log} = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad [12]$$

In the second coordinate system, inflation is characterised by a relative increase in the price level, and unemployment - by a relative increase in the maximum possible employment:

$$x_{rate} = \frac{\max E_t}{E_t} - 1 = \frac{U_t}{L_t - U_t} \quad [13]$$

$$y_{rate} = \frac{P_t}{P_{t-1}} - 1 = \frac{\Delta P_t}{P_{t-1}} \quad [14]$$

The variability of the abscissa from zero to infinity is the advantage of both coordinate systems.

3.2. Methodology

In the proposed paper, a non-parametric approach, namely the DEA method, is used to assess the international state of countries. This method consists in constructing a convex hull of the set of countries under study and determining its effective part. In practice, this means that on this conditional frontier, the best value of one indicator corresponds to a certain value of the other, and vice versa. Given that smaller values of both coordinates - inflation and unemployment indicators - are preferable, lower left part of the convex hull is its efficient frontier. By analogy with the parametric "inflation - unemployment" curve, this line will hereinafter be called the international Phillips DEA line. However, it is worth noting that, like the effective DEA frontiers for other macroeconomic indicators, this line is a tool for normative analysis, not forecasting. If parametric models with Phillips curves make it possible to predict how one of the indicators will change when another changes, DEA models make it possible to give an objective assessment of the lag of "outsider" countries from "leader" ones. The international Phillips DEA line is a broken polygonal chain, in the vertices of which there are the countries with the best combinations of inflation and unemployment indicators. Each of its intermediate segments is a set of mean weighted states of the countries located at its ends. The country with the lowest unemployment rate will be the upper left point of the international Phillips DEA line, and the country with the lowest inflation rate will be the lower right point.

As in the case of parametric models, in the proposed DEA model it is possible to determine the optimal state on the efficiency frontier. For this, the international social loss function must be convex in the opposite direction relative to the efficiency frontier. If the loss function is a closed curve, its centre must be located outside the convex hull of the states of the countries. The number of such states is infinite, but among them there is one objectively selected state. This is a virtual state, the coordinates of which are the best actual values of inflation and unemployment indicators in the studied set. The virtual point $\mathcal{I}$ with minimum values of both indicators can be called the "ideal" state of the studied set of countries (Zagoruiko & Petkova, 2022). It is this state that has been chosen as the centre of elliptic curves of international social losses.

Thus, unlike econometric models of national economies, in the proposed model the best possible state of the selected set of countries is chosen as a guideline for economic policy. In practice, this means that a country that seeks to reach the international efficiency frontier should take into account not only its own existing interdependence of inflation and unemployment, but also the experience of countries that have minimised at least one of the two indicators.

In the general case, the social loss function does not necessarily have to be quadratic. As already shown in the literature review, a closed elliptic-type curve with power indices other than 2 can be chosen as such a function. In this case, power indices to which unemployment and inflation levels are raised can be different. With this approach, power indices are converted into parameters. This will require a certain technique for calculating them, and therefore additional assumptions.

Hyperbolic-type curves are an alternative to closed social loss curves. Two virtual states - "ideal" one with the lowest levels of inflation and unemployment and "terrible" one with the highest levels - could be the foci of such curves. A hyperbolic curve of international social losses means that countries are interested not only in getting closer to the "leaders", but also in moving away from the rest of the "outsiders". This would make the DEA model less dependent on the state of "leader" countries, but also more complex.

Given the mentioned problems, the proposed study uses the simplest and at the same time the most popular function - the quadratic social loss function of the elliptic type. Its advantage is that it agrees well with the measurements of Euclidean distances between the states of the studied countries.

Thus, the proposed international social loss function is the sum of the squares of the deviations of the actual state of the country from the current "ideal" state. Although mathematically these deviations are dimensionless quantities, they have different meanings from the point of view of economic policy. Obviously, a deviation from the minimum inflation rate is not equivalent to a deviation from the minimum unemployment rate. It is desirable to normalise them by dividing by a similar deviation of a certain mean state $\mathcal{M}$.

Different ways of constructing such a mean state are possible. One possible way is to calculate arithmetic mean levels of inflation and unemployment for the entire set of countries. This would make the model more stable, but also more inert - one that reflects less the achievements of "leader" countries and more the states of "outsider" countries. According to the authors, it is more logical from the point of view of the DEA approach to use information that characterises only "leader" countries. However, the states of the countries that form the international Phillips DEA line are not equivalent. The first and last countries on the international efficiency frontier have achieved the best value of only one of the two indicators. At the same time, countries located on intermediate segments have a more balanced combination of indicators. To take into account these features, it is advisable to assign certain weighting coefficients to "leader" countries. In the DEA model, such coefficients should be non-parametric. This can be done in the following geometric way. First, the midpoint of each link of the efficiency frontier is determined. Then these points are sequentially connected by segments, for which the midpoints are again determined. This procedure is repeated until a single point - the $\mathcal{M}(\overline{x_{eff}};\overline{y_{eff}})$ state - is found.

Algebraically, this procedure means the sequential calculation of arithmetic mean coordinates of the points at the ends of constructed segments. The resulting weighting coefficients are binomial. Namely, they are fractions in which the numerator is the binomial coefficient and the denominator is the sum of the numerators.

Thus, the sum of such weighting coefficients will be equal to one:

$$\frac{1}{2^{N-1}}\sum_{n=1}^{N}\binom{N-1}{n-1} = \frac{1}{2^{N-1}}\sum_{n=1}^{N}\frac{(N-1)!}{(n-1)!\,(N-n)!} = 1 \qquad [15]$$

where n, N are the country number and the total number of countries on the international efficiency frontier.

As a result, the equation of the social loss curve of the i-th country takes the form:

$$\mathbb{L}(\tilde{x};\tilde{y}) = \left(\frac{x - x_{min}}{\overline{x_{eff}} - x_{min}}\right)^2 + \left(\frac{y - y_{min}}{\overline{y_{eff}} - y_{min}}\right)^2 = \mathbb{L}(C_i) \qquad [16]$$

where $\tilde{x}, \tilde{y}$ are normalised coordinates; x_{min}, y_{min} are the coordinates of the "ideal" state; x_{eff}, y_{eff} are the coordinates of countries' states on the efficiency frontier; C_i is the actual state of the i-th country. In the normalised $\tilde{x}\mathcal{I}\tilde{y}$ coordinate system, the value of the social loss function is equal to the square of the radius vector of the corresponding curve.

As follows from this equation $\left(\overline{x_{eff}} - x_{min}\right)^{-2}$ and $\left(\overline{y_{eff}} - y_{min}\right)^{-2}$ factors are weighting coefficients of deviations from the "ideal" state. The further the $\mathcal{M}(\overline{x_{eff}};\overline{y_{eff}})$ point is from the "ideal" state, the smaller these weighting coefficients will be. In the general case, such weighting coefficients will be unequal.

Formally, a similar procedure for constructing the $\mathcal{M}$ point can be applied to other types of means. However, certain economic and mathematical problems arise in this case. Thus, compared to the arithmetic mean, the geometric mean of the ends of the segment will be shifted in the direction of smaller values. Since the segments of the international Phillips DEA line have a negative slope, the $\mathcal{M}_{geom}$ point will be outside the convex hull. On the contrary, the quadratic mean of the ends of the segment will be shifted towards larger values. The $\mathcal{M}_{quadr}$ point constructed in this way will be located within the convex hull, but will be further from it than $\mathcal{M}_{arith}$. Both of these types of means do not have such a simple geometric interpretation as the arithmetic mean. There are two more specific problems caused by their use. The first problem is that the geometric mean is calculated only for positive non-zero values. This means that its use requires the choice of a different coordinate system - for example, an exponential one. A similar problem arises in the case of using the quadratic mean. Algebraically, it can also be calculated for negative values. However, the “loss” of the negative sign after squaring also means the loss of economic meaning. As for $\mathcal{M}_{arith}$, it could be constructed by successively dividing the segments in a proportion other than 1:1. However, in practice, this would mean that already at the first stage of the calculations, one of the two countries would receive greater priority than the other.

It is worth noting that, like the international Phillips DEA line, the proposed international social loss function is non-parametric. It is built on the basis of only those states that are mapped in selected coordinate systems. It is brought closer to the parametric social loss function by the selected mathematical form, as well as the uniqueness for all possible shifts of Phillips curves. In the econometric (parametric) “inflation - unemployment” model, the national social loss function is constant and weighs the national policy goals from the point of view of domestic economic state. In the DEA model, the international social loss function is also constant, but reflects and weighs the actual states of countries from the point of view of their international position. The universality of the international social loss function makes it possible to assess the actual states of all countries with a single “measure”. At the same time, different countries can and should seek their own trade-offs between national and international social losses.

In the proposed model, the international optimum $\mathcal{O}_{inter}$ is the point of tangency of the social loss curve with the international efficiency frontier. Algebraically, it is defined as the solution of the conditional minimum problem:

$$\begin{cases} \mathbb{L}(\mathrm{x},\mathrm{y}) \longrightarrow \min \\ \left(\dfrac{x - x_{\Xi}}{x_{\Psi} - x_{\Xi}}\right) - \left(\dfrac{y - y_{\Xi}}{y_{\Psi} - y_{\Xi}}\right) = 0 \end{cases} \qquad [17]$$

where Ξ, Ψ are countries at the ends of the segment, which is touched by the social loss curve. In the optimal state, the following condition is fulfilled:

$$\frac{(x - x_{min})}{\left(\overline{x_{eff}} - x_{min}\right)^2} : \frac{(y - y_{min})}{\left(\overline{y_{eff}} - y_{min}\right)^2} = -\frac{(y_{\Psi} - y_{\Xi})}{(x_{\Psi} - x_{\Xi})} \qquad [18]$$

This condition is met not only by the point of the international optimum, but also by the point of the “ideal” $\mathcal{I}(x_{min};\ y_{min})$ state. The $\mathcal{I}\mathcal{O}_{inter}$ straight line, passing through these two points, sets a direction that can be considered as allocated. If national Phillips DEA lines are formed by parallel transfer of the international efficiency frontier along this direction, then all points of national optima will lie on this straight line. On the line of optima, the structure of social losses is constant.

Mathematically, this method is the simplest one. However, a more weighty theoretical argument can be made in its favour. The international efficiency frontier, constructed on the basis of one year's data, can be interpreted as an actual short-run Phillips DEA line. Under this interpretation, the lines formed by its parallel transfer will be its virtual maps. This interpretation of the proposed DEA model turns it into a kind of analogue of those parametric models in which the linear short-run Phillips function shifts in parallel according to the change in the expected inflation rate.

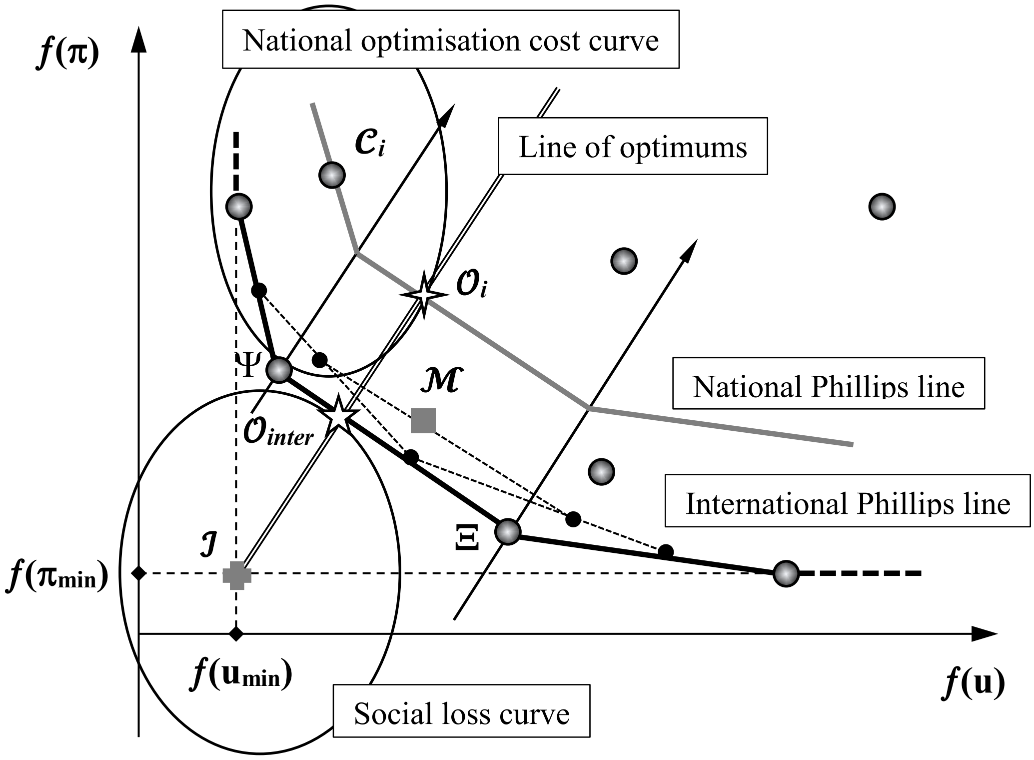

Figure 1. Optimisation of countries' states using the elliptic social loss function. *Source:* I. Zagoruiko

It is clear that national Phillips DEA lines constructed in this way will not coincide with national curves constructed in an econometric way. Their nature and purpose are different. They characterise the current position of "outsider" countries relative to "leader" ones. "Outsider" countries can somehow take into account their position relative to the efficiency frontier, or they can ignore it. In practice, the country's movement towards the national DEA optimum, and even more so the movement towards the international optimum, is a long process that will require complex trade-offs between existing macroeconomic dependencies and the optimisation of its international position. Most likely, it will require serious changes in the entire economic policy of the state and its legislation. Such problems require a separate study and therefore are not the subject of the proposed paper.

The chosen method of constructing national "Phillips polygonal chains" simplifies the calculation of the value of their parallel transfer. It is convenient to define it as the Euclidean distance between the point of the national optimum $\mathcal{O}_i$ and the point of the "ideal" state $\mathcal{I}$:

$$\rho(\mathcal{O}_i, \mathcal{I}) = \sqrt{\left(x_{i,opt} - x_{min}\right)^2 + \left(y_{i,opt} - y_{min}\right)^2} \quad [19]$$

The ratio of the national distance to the distance of the international optimum will be the translation coefficient (parallel transfer) of Phillips DEA lines:

$$\vartheta_i = \frac{\rho(\mathcal{O}_i, \mathcal{I})}{\rho(\mathcal{O}_{inter}, \mathcal{I})} \quad [20]$$

For the international efficiency frontier, the translation coefficient is equal to one:

$$\vartheta(\mathcal{O}_{inter}) = 1 \quad [21]$$

From a practical point of view, the proposed model for assessing the international state of a country means the following. If an "outsider" country chooses a strategy of moving along the national Phillips DEA line, then its optimal state will be worse than the international optimum by ϑ_i times. If, having reached the national optimum, a country chooses a course towards the international optimum, then the indicators characterising its inflation and unemployment will have to be proportionally reduced according to ϑ_i coefficient.

As in the case of the international social loss curve, national DEA lines can be constructed in other ways. Thus, national DEA lines can be obtained by stretching the international efficiency frontier along the rays emanating from the point of the "ideal" state. The links of each subsequent DEA line will become larger and larger. As a result, each subsequent "outsider" country will be more and more likely to fall on the optimal segment of its national line. Since Phillips DEA lines are convex, the new optimal segment will be located above and to the right of the similar optimal segment in the model with the efficiency frontier shifted along the $\mathcal{J}\mathcal{O}_{inter}$ line, and therefore will be worse. Similar problems may arise if national DEA lines are shifted in parallel, but along a different direction, for example $\mathcal{J}\mathcal{M}$.

A more radical way to modify the proposed model is to sequentially construct efficiency frontiers of subsequent, lower orders. According to this method, countries on the lower left part of the convex hull are first excluded from the set of countries. For the remaining countries, their own efficiency frontier is constructed. By repeating this procedure, it is possible to construct increasingly worse efficiency frontiers. The shapes of such broken lines will no longer be similar to the first efficiency frontier, and several countries may be located on each of them. In this modified model, it is not possible to unambiguously determine the distance of a country to the first-order efficiency frontier. Since the mutual position of "outsider" countries is more mobile than the position of "leader" ones, the DEA model itself becomes less stable.

The international social loss function introduced above also makes it possible to determine international costs of countries to optimise their state:

$$\mathbb{L}(\Delta\tilde{x}_i; \Delta\tilde{y}_i) = \left(\frac{x_{i,act} - x_{i,opt}}{\overline{\overline{x_{eff}}} - x_{min}}\right)^2 + \left(\frac{y_{i,act} - y_{i,opt}}{\overline{\overline{y_{eff}}} - y_{min}}\right)^2 = \tilde{\rho}^2(\mathcal{C}_i; \mathcal{O}_i) \qquad [22]$$

where $x_{i,act}$, $y_{i,act}$ are the coordinates of the actual state, and $x_{i,opt}$, $y_{i,opt}$ are the coordinates of the optimal state $\mathcal{O}_i$ of the i-th country on its Phillips DEA line. In a normalised $\tilde{x}\mathcal{J}\tilde{y}$ coordinate system, optimisation costs are equal to the square of the distance between actual and optimal states.

The ratio of the value of social loss reduction to the value of optimisation costs characterises the degree of efficiency of changing the state of the country:

$$\epsilon_i(\mathcal{O}_i) = \frac{\Delta\mathbb{L}(\tilde{x}_i; \tilde{y}_i)}{\mathbb{L}(\Delta\tilde{x}_i; \Delta\tilde{y}_i)} = \frac{\tilde{\rho}^2(\mathcal{J}, \mathcal{C}_i) - \tilde{\rho}^2(\mathcal{J}, \mathcal{O}_i)}{\tilde{\rho}^2(\mathcal{C}_i, \mathcal{O}_i)} \geq 1 \qquad [23]$$

where $\epsilon_i(\mathcal{O}_i)$ is the national coefficient of optimisation efficiency on the constant "Phillips polygonal chain", $\tilde{\rho}$ is the distance between states in the normalised coordinate system. By construction, the optimisation efficiency coefficient ϵ is not less than one.

In addition to movement along its own "Phillips polygonal chain", the country can choose an alternative strategy - a transition immediately to the international optimum $\mathcal{O}_{inter}(\min x_{opt}; \min y_{opt})$. In this case, its optimisation costs will be equal to

$$\mathbb{L}(\Delta\tilde{x}_i;\Delta\tilde{y}_i)=\left(\frac{x_{i,act}-\min x_{opt}}{\overline{x_{eff}}-x_{min}}\right)^2+\left(\frac{y_{i,act}-\min y_{opt}}{\overline{y_{eff}}-y_{min}}\right)^2 \quad [24]$$

For this strategy, it is also possible to determine the national optimisation efficiency coefficient

$$\epsilon_i(\mathcal{O}_{inter})=\frac{\tilde{\rho}^2(\mathcal{I},\mathcal{C}_i)-\tilde{\rho}^2(\mathcal{I},\mathcal{O}_{inter})}{\tilde{\rho}^2(\mathcal{C}_i,\mathcal{O}_{inter})}\geq 1 \quad [25]$$

The ratio of these two efficiency coefficients determines the degree of feasibility of the first strategy - the step-by-step optimisation strategy:

$$\varepsilon_i=\frac{\epsilon_i(\mathcal{O}_i)}{\epsilon_i(\mathcal{O}_{inter})} \quad [26]$$

If ε_i is greater than one, then it is better for the country to first reach the national optimum and only then - the international one. Otherwise, the strategy of transition directly to the international optimum should be preferred.

To conclude this theoretical section, it is worth giving one more argument in favour of choosing the elliptical form of the international social loss function. As follows from the above, the definition of international costs of countries to optimise their state is well consistent with this form. In the case of using a hyperbolic or other function of international social losses, this consistency will disappear.

4. Empirical results and preliminary analyses

The proposed model of the optimal state of countries was tested on the basis of European Union data (Eurostat, 2024a and 2024b). The mean annual Harmonised Index of Consumer Prices (HICP) was chosen as the initial inflation indicator, and the number of unemployed people aged 15-74 as a percentage of the labour force was chosen as the initial unemployment indicator (according to the International Labour Organisation methodology).

Initial statistical data are presented in Table 1. For the purpose of further comparative analysis, the European Union was considered as another (conditional) country.

It is noteworthy that in 2023, none of the countries reached the 2% inflation rate, which the European Central Bank considers to be the target indicator in the medium term (ECB, 2025; Benigno et al., 2023). The lowest inflation rate was observed in Belgium (2.3%), and the lowest unemployment rate - in the Czechia (2.6%). On the “unemployment rate - inflation rate” plane, these two countries represented, respectively, the lower right and upper left points of the general efficiency frontier - the international Phillips DEA line. The middle segment of this polygonal chain was formed by Germany (3.1%; 6.0%) and the Netherlands (3.6%; 4.1%). Hungary (4.1%; 17.0%) and Spain (12.2%; 3.4%) made up the inefficient (opposite) part of the convex hull.

According to the described methodology, initial statistical data were transformed into the coordinates of countries' actual states on two other planes - the plane of logarithms of indices and the plane of relative increments. These transformations did not change the convex hull vertices. For almost all countries, the corresponding coordinates on all three planes coincided with an accuracy to the second decimal place. In the case of the "ideal" state, these coordinates were equal to (0.0263; 0.0227) on the logarithmic plane and (0.0267; 0.0230) on the increment plane.

Table 1. Actual states and translation coefficient of Phillips DEA lines of European Union countries in 2023.

Country	Plane of initial indicators		Plane of logarithms of indices			Plane of relative increments		
	Unemployment rate, %	Inflation rate, %	Logarithm of potential employment index	Logarithm of actual price index	Phillips line translation coefficient	Potential increase in employment	Actual increase in price level	Phillips line translation coefficient
	$100u$	100π	$-\ln(1-u)$	$\ln(1+\pi)$	ϑ	$\frac{u}{1-u}$	π	ϑ
EU - 27	6.1	6.40	0.0629	0.0620	2.4005	0.0650	0.0640	2.4124
Austria	5.1	7.7	0.0523	0.0742	2.4898	0.0537	0.0770	2.5124
Belgium	5.5	2.3	0.0566	0.0227	1.0000	0.0582	0.0230	1.0000
Bulgaria	4.3	8.6	0.0440	0.0825	2.2667	0.0449	0.0860	2.2890
Croatia	6.1	8.4	0.0629	0.0807	2.9816	0.0650	0.0840	3.0155
Cyprus	5.8	3.9	0.0598	0.0383	1.5713	0.0616	0.0390	1.5702
Czechia	2.6	12.0	0.0263	0.1133	1.0000	0.0267	0.1200	1.0000
Denmark	5.1	3.4	0.0523	0.0334	1.2183	0.0537	0.0340	1.2156
Estonia	6.4	9.1	0.0661	0.0871	3.2700	0.0684	0.0910	3.3154
Finland	7.2	4.3	0.0747	0.0421	2.1006	0.0776	0.0430	2.1076
France	7.3	5.7	0.0758	0.0554	2.5462	0.0787	0.0570	2.5601
Germany	3.1	6.0	0.0315	0.0583	1.0000	0.0320	0.0600	1.0000
Greece	11.1	4.2	0.1177	0.0411	3.2447	0.1249	0.0420	3.3078
Hungary	4.1	17.0	0.0419	0.1570	2.8630	0.0428	0.1700	2.9109
Ireland	4.3	5.2	0.0440	0.0507	1.5273	0.0449	0.0520	1.5292
Italy	7.7	5.9	0.0801	0.0573	2.7234	0.0834	0.0590	2.7421
Latvia	6.5	9.1	0.0672	0.0871	3.2992	0.0695	0.0910	3.3452
Lithuania	6.9	8.7	0.0715	0.0834	3.3018	0.0741	0.0870	3.3442
Luxembourg	5.2	2.9	0.0534	0.0286	1.0958	0.0549	0.0290	1.0938
Malta	3.5	5.6	0.0356	0.0545	1.1996	0.0363	0.0560	1.2004
Netherlands	3.6	4.1	0.0367	0.0402	1.0000	0.0373	0.0410	1.0000
Poland	2.8	10.9	0.0284	0.1035	1.1077	0.0288	0.1090	1.1065
Portugal	6.5	5.3	0.0672	0.0516	2.1929	0.0695	0.0530	2.1992
Romania	5.6	9.7	0.0576	0.0926	3.2084	0.0593	0.0970	3.2607
Slovakia	5.8	11.0	0.0598	0.1044	3.6341	0.0616	0.1100	3.7113
Slovenia	3.7	7.2	0.0377	0.0695	1.6160	0.0384	0.0720	1.6240
Spain	12.2	3.4	0.1301	0.0334	3.3446	0.1390	0.0340	3.4333
Sweden	7.7	5.9	0.0801	0.0573	2.7234	0.0834	0.0590	2.7421

Source: compiled by the authors based on Eurostat data (2024a and 2024b) and own calculations.

***Note:** the plane of initial indicators displays the initial (untransformed) data; the plane of logarithms of indices displays the data transformed according to formulas [11] and [12]; the plane of relative increments displays the actual inflation rate and the unemployment indicator calculated according to formula [13].

Since the international "Phillips polygonal chain" was formed by four countries, weighting coefficients used to calculate the binomial mean were equal to 1/8 for the Czechia and Belgium and

3/8 for countries in the middle segment - Germany and the Netherlands. The point of the binomial mean $\mathcal{M}$ was located above the bisector of the right angle with the vertex at the point of the "ideal" state $\mathcal{I}$. On the logarithmic plane, it had coordinates $\mathcal{M}_{log}(0.0359; 0.0539)$, and on the plane of increments - $\mathcal{M}_{rate}(0.0366; 0.0558)$. Thus, on the logarithmic plane, the canonical equation of the international social loss function had the form:

$$\mathbb{L}_{log} = \left(\frac{x - 0.0263}{0.0359 - 0.0263}\right)^2 + \left(\frac{y - 0.0227}{0.0539 - 0.0227}\right)^2 = const \qquad [27]$$

where x is the logarithm of the potential employment index and y is the logarithm of the price index. On the plane of increments, a similar equation was

$$\mathbb{L}_{rate} = \left(\frac{x - 0.0267}{0.0366 - 0.0267}\right)^2 + \left(\frac{y - 0.0230}{0.0558 - 0.0230}\right)^2 = const \qquad [28]$$

where x is the relative potential increase in employment, and y is the relative increase in the actual price level (inflation rate).

As these equations show, the deviation of the point of the binomial mean $\mathcal{M}$ from the point of the "ideal" state $\mathcal{I}$ was much larger along the ordinate axis than along the abscissa axis. As a result, social loss curves acquired an eccentricity approaching one: $e_{log} = 0.9517$ on the logarithmic plane and $e_{rate} = 0.9530$ on the plane of increments. The extremely elongated vertical shape of these ellipses was due to the fact that the difference in inflation rates in the Czechia and Belgium far exceeded the difference in their unemployment rates. From a theoretical point of view, this means that in the obtained social loss function, exceeding the minimum (Belgian) inflation rate had a smaller weighting coefficient than exceeding the minimum (Czech) unemployment rate. Because of this feature, the point of the international optimum was on the middle segment of the "Phillips polygonal chain" - closer to the point of the Netherlands than to the point of Germany: $\mathcal{O}_{inter,log}(0.0345; 0.0476)$ - on the logarithmic plane and $\mathcal{O}_{inter,rate}(0.0351; 0.0489)$ - on the plane of increments. From a practical point of view, this means that large values of the eccentricity of social loss curves can significantly shift points of DEA optima - international and national ones. However, if a certain link of the international efficiency frontier, located in the middle, is significantly extended in the same direction as loss curves, the optimum point will only "slide" along it.

According to the proposed approach, all national optima should be located on a straight line passing through the point of the "ideal" state $\mathcal{I}$ and the point of the international optimum $\mathcal{O}_{inter}$. Middle segments of national "Phillips polygonal chains" are limited by parallel lines drawn at the points of Germany and the Netherlands. The coefficients of the equations of these straight lines are presented in Table 2.

Table 2. Coefficients of the equations of international efficiency frontier segments and its secants.

Plane	Coefficient	Efficiency frontier segment			Efficiency frontier secant		
		Czechia - Germany	Germany– Netherlands	Netherlands- Belgium	At the point of Germany	At the point of optimum	At the point of Netherlands
Logarithmic plane	a_0	0.3952	0.1684	0.0723	−0.0372	−0.0571	−0.0710
	a_1	−10.6981	−3.4962	−0.8762	3.0325	3.0325	3.0325
Plane of increments	a_0	0.4223	0.1736	0.0732	−0.0383	−0.0590	−0.0737
	a_1	−11.3257	−3.5496	−0.8630	3.0712	3.0712	3.0712

Source: authors' own calculations.

The calculated coordinates of optimal states are presented in Table 3. The obtained coordinates of national optimum points were used to determine the corresponding translation coefficients of national "Phillips polygonal chains". The mean translation coefficient $\overline{\vartheta}$ for 27 EU countries was 2.2047 on the logarithmic plane and 2.2273 on the plane of increments. These values are somewhat smaller than the corresponding values for the EU as a whole.

At the next stage of the study, social losses were calculated and analysed (Table 4). It is noteworthy that for both the EU-27 and the average member country, the specific weight of actual unemployment losses was several times higher than the specific weight of similar inflation losses. On average, optimisation of the states of the studied countries more than halved total social losses and levelled their structure. Namely, on the logarithmic plane, the optimal share of unemployment losses was 53.55%, and on the plane of increments - 53.61% of the total value. On average, the optimisation efficiency coefficient slightly exceeded 2 and was noticeably inferior to the value for the conditional EU-27 country.

The found values of total social losses were used to compare the results of the study using the method of logarithms of indices and the method of relative increments. It turned out that there was a close correlation between total losses calculated using two different methods. For the set of actual states of countries, supplemented by EU-27 points and the "ideal" state, this equation had the form

$$\mathbb{L}_{rate}(\mathcal{C}_i) = -0.823 + 1.0872\mathbb{L}_{log}(\mathcal{C}_i) \quad R^2 = 0.9996 \qquad [29]$$

where $\mathbb{L}_{log}$ is total social losses according to the logarithmic version of the model, and $\mathbb{L}_{rate}$ is the losses calculated using the version of relative increments. For a similar set of optimal states, the regression line was even closer to the bisector:

$$\mathbb{L}_{rate}(\mathcal{O}_i) = -0.108 + 1.0231\mathbb{L}_{log}(\mathcal{O}_i) \quad R^2 = 0.9997 \qquad [30]$$

Table 3. Optimal states of European Union countries in 2023.

Country	Plane of logarithms of indices		Plane of relative increments		Plane of initial indicators: Logarithmic model		Plane of initial indicators: Model of increments	
	Logarithm of potential employment index	Logarithm of price index	Potential increase in employment	Increase in price level	Unemployment rate, %	Inflation rate, %	Unemployment rate, %	Inflation rate, %
	$-\ln(1-u)$	$\ln(1+\pi)$	$\frac{u}{1-u}$	π	$100u$	100π	$100u$	100π
EU - 27	0.0460	0.0824	0.0470	0.0854	4.50	8.59	4.49	8.54
Austria	0.0468	0.0846	0.0479	0.0880	4.57	8.83	4.57	8.80
Belgium	0.0345	0.0476	0.0351	0.0489	3.40	4.88	3.39	4.89
Bulgaria	0.0449	0.0791	0.0460	0.0823	4.39	8.23	4.40	8.23
Croatia	0.0508	0.0969	0.0521	0.1011	4.95	10.17	4.95	10.11
Cyprus	0.0392	0.0618	0.0399	0.0636	3.85	6.38	3.84	6.36
Czechia	0.0345	0.0476	0.0351	0.0489	3.40	4.88	3.39	4.89
Denmark	0.0363	0.0530	0.0369	0.0545	3.57	5.45	3.56	5.45
Estonia	0.0532	0.1040	0.0546	0.1088	5.18	10.96	5.18	10.88
Finland	0.0436	0.0750	0.0445	0.0776	4.26	7.78	4.26	7.76
France	0.0472	0.0860	0.0483	0.0893	4.61	8.98	4.60	8.93
Germany	0.0345	0.0476	0.0351	0.0489	3.40	4.88	3.39	4.89
Greece	0.0529	0.1034	0.0546	0.1086	5.16	10.89	5.18	10.86
Hungary	0.0498	0.0939	0.0512	0.0984	4.86	9.85	4.87	9.84
Ireland	0.0389	0.0607	0.0396	0.0626	3.81	6.26	3.81	6.26
Italy	0.0487	0.0904	0.0498	0.0940	4.75	9.47	4.74	9.40
Latvia	0.0534	0.1048	0.0549	0.1096	5.20	11.04	5.20	10.96
Lithuania	0.0534	0.1048	0.0549	0.1096	5.20	11.05	5.20	10.96
Luxembourg	0.0353	0.0500	0.0359	0.0513	3.47	5.13	3.47	5.13
Malta	0.0362	0.0526	0.0368	0.0541	3.55	5.40	3.55	5.41
Netherlands	0.0345	0.0476	0.0351	0.0489	3.40	4.88	3.39	4.89
Poland	0.0354	0.0503	0.0360	0.0516	3.48	5.16	3.48	5.16
Portugal	0.0443	0.0773	0.0452	0.0799	4.34	8.03	4.33	7.99
Romania	0.0526	0.1025	0.0542	0.1074	5.13	10.79	5.14	10.74
Slovakia	0.0561	0.1131	0.0580	0.1191	5.46	11.97	5.48	11.91
Slovenia	0.0396	0.0629	0.0404	0.0650	3.88	6.49	3.88	6.50
Spain	0.0538	0.1059	0.0556	0.1119	5.23	11.17	5.27	11.19
Sweden	0.0487	0.0904	0.0498	0.0940	4.75	9.47	4.74	9.40

Source: authors' own calculations.

Table 4. Social losses of European Union countries in 2023.

Country	According to the logarithmic model				According to the model of relative increments			
	Actual losses		Optimal total losses	Reduction of total losses, %	Actual losses		Optimal total losses	Reduction of total losses, %
	Total losses	Unemployment losses, %			Total losses	Unemployment losses, %		
	$\mathbb{L}(C_i)$	$\frac{\mathbb{L}_u(C_i)}{\mathbb{L}(C_i)}$	$\mathbb{L}(O_i)$	$\frac{\Delta\mathbb{L}}{\mathbb{L}(C_i)}$	$\mathbb{L}(C_i)$	$\frac{\mathbb{L}_u(C_i)}{\mathbb{L}(C_i)}$	$\mathbb{L}(O_i)$	$\frac{\Delta\mathbb{L}}{\mathbb{L}(C_i)}$
EU - 27	16.19	90.19	7.88	51.30	16.45	90.47	7.84	52.36
Austria	10.09	73.04	8.48	15.96	10.15	73.23	8.50	16.28
Belgium	9.96	100.0	1.37	86.26	10.09	100.0	1.35	86.65
Bulgaria	7.05	47.93	7.03	0.32	7.08	47.74	7.06	0.34
Croatia	18.05	80.89	12.16	32.61	18.35	81.10	12.25	33.27
Cyprus	12.41	98.01	3.38	72.79	12.60	98.11	3.32	73.65
Czechia	8.44	0.00	1.37	83.79	8.77	0.00	1.35	84.65
Denmark	7.49	98.43	2.03	72.88	7.55	98.51	1.99	73.63
Estonia	21.52	80.21	14.63	32.03	21.97	80.38	14.80	32.61
Finland	25.90	98.51	6.04	76.69	26.70	98.60	5.98	77.59
France	27.76	96.04	8.87	68.05	28.62	96.23	8.83	69.16
Germany	1.59	18.20	1.37	13.77	1.56	18.27	1.35	13.75
Greece	91.23	99.62	14.40	84.21	98.28	99.66	14.74	85.01
Hungary	21.16	12.41	11.21	47.00	22.77	11.51	11.41	49.88
Ireland	4.18	80.79	3.19	23.71	4.17	81.17	3.15	24.38
Italy	32.76	96.25	10.15	69.03	33.92	96.44	10.13	70.14
Latvia	22.46	81.04	14.89	33.70	22.95	81.22	15.07	34.33
Lithuania	26.00	85.44	14.91	42.65	26.67	85.68	15.06	43.53
Luxembourg	8.01	99.56	1.64	79.50	8.09	99.59	1.61	80.09
Malta	1.98	47.55	1.97	0.36	1.95	47.86	1.94	0.33
Netherlands	1.47	78.78	1.37	7.17	1.46	79.24	1.35	7.43
Poland	6.74	0.68	1.68	75.11	6.94	0.65	1.65	76.24
Portugal	19.06	95.49	6.58	65.49	19.48	95.69	6.51	66.56
Romania	15.68	68.03	14.08	10.21	15.93	67.94	14.32	10.08
Slovakia	19.01	63.98	18.07	4.98	19.42	63.66	18.55	4.48
Slovenia	3.66	38.45	3.57	2.29	3.64	38.44	3.55	2.32
Spain	117.48	99.90	15.30	86.97	128.2	99.91	15.88	87.62
Sweden	32.76	96.25	10.15	69.03	33.92	96.44	10.13	70.14
Average	**21.26**	**86.23**	**7.77**	**63.43**	**22.27**	**86.45**	**7.85**	**64.77**

Source: authors' own calculations.

The study ended with the calculation of costs for countries to achieve national and international optima and efficiency coefficients of these costs (Table 5).

Table 5. Efficiency of optimisation of European Union economies in 2023.

Country	Plane of logarithms of indices					Plane of relative increments				
	Optimum				Ratio of efficiency coefficients	Optimum				Ratio of efficiency coefficients
	national, $\mathcal{O}_i$		international, $\mathcal{O}_{inter}$			national, $\mathcal{O}_i$		international, $\mathcal{O}_{inter}$		
	Optimisation costs, %	Efficiency coefficient	Optimisation costs, %	Efficiency coefficient		Optimisation costs, %	Efficiency coefficient	Optimisation costs, %	Efficiency coefficient	
	$\frac{\mathbb{L}(\Delta\mathcal{C})}{\mathbb{L}(\mathcal{C}_i)}$	ϵ_i	$\frac{\mathbb{L}(\Delta\mathcal{C})}{\mathbb{L}(\mathcal{C}_i)}$	ϵ_i	ε_i	$\frac{\mathbb{L}(\Delta\mathcal{C})}{\mathbb{L}(\mathcal{C}_i)}$	ϵ_i	$\frac{\mathbb{L}(\Delta\mathcal{C})}{\mathbb{L}(\mathcal{C}_i)}$	ϵ_i	ε_i
EU - 27	21.91	2.34	55.63	1.65	1.42	22.48	2.33	56.30	1.63	1.43
Austria	4.49	3.55	41.44	2.09	1.70	4.57	3.56	41.96	2.07	1.72
Belgium	59.49	1.45	59.49	1.45	1.00	59.85	1.45	59.85	1.45	1.00
Bulgaria	0.32	1.00	31.45	2.56	0.39	0.34	1.00	31.95	2.53	0.39
Croatia	10.42	3.13	54.93	1.68	1.86	10.63	3.13	55.57	1.67	1.88
Cyprus	41.59	1.75	56.53	1.57	1.11	42.27	1.74	57.14	1.56	1.11
Czechia	61.33	1.37	61.33	1.37	1.00	61.98	1.37	61.98	1.37	1.00
Denmark	42.61	1.71	48.90	1.67	1.02	43.18	1.71	49.41	1.66	1.03
Estonia	9.91	3.23	58.02	1.61	2.00	10.08	3.24	58.68	1.60	2.02
Finland	45.15	1.70	68.07	1.39	1.22	45.95	1.69	68.77	1.38	1.22
France	35.55	1.91	67.07	1.42	1.35	36.38	1.90	67.81	1.41	1.35
Germany	13.77	1.00	13.77	1.00	1.00	13.75	1.00	13.75	1.00	1.00
Greece	54.40	1.55	82.58	1.19	1.30	55.30	1.54	83.32	1.18	1.30
Hungary	22.60	2.08	60.92	1.54	1.35	24.23	2.06	62.67	1.50	1.37
Ireland	9.21	2.57	23.31	2.89	0.89	9.49	2.57	23.70	2.86	0.90
Italy	36.36	1.90	69.44	1.38	1.38	37.23	1.88	70.19	1.37	1.38
Latvia	10.69	3.15	58.93	1.59	1.98	10.88	3.16	59.60	1.58	2.00
Lithuania	15.52	2.75	62.31	1.52	1.81	15.88	2.74	63.01	1.51	1.82
Luxembourg	50.29	1.58	53.00	1.56	1.01	50.79	1.58	53.45	1.56	1.01
Malta	0.36	1.00	3.12	9.86	0.10	0.33	1.00	3.11	9.92	0.10
Netherlands	7.17	1.00	7.17	1.00	1.00	7.43	1.00	7.43	1.00	1.00
Poland	51.09	1.47	53.66	1.49	0.99	51.81	1.47	54.38	1.48	0.99
Portugal	33.49	1.96	61.11	1.52	1.29	34.25	1.94	61.81	1.51	1.29
Romania	2.37	4.31	50.31	1.81	2.37	2.32	4.34	50.93	1.80	2.41
Slovakia	1.16	4.29	53.85	1.72	2.49	1.07	4.18	54.54	1.71	2.45
Slovenia	2.29	1.00	16.49	3.79	0.26	2.32	1.00	16.74	3.76	0.27
Spain	58.67	1.48	84.91	1.16	1.27	59.45	1.47	85.63	1.16	1.28
Sweden	36.36	1.90	69.44	1.38	1.38	37.23	1.88	70.19	1.37	1.38
Average	**26.54**	**2.07**	**50.80**	**1.97**	**1.05**	**38.46**	**2.06**	**68.97**	**1.36**	**1.51**

Source: authors' own calculations.

5. Discussion of the results of the study for individual EU countries

As noted above, the international frontier of the best combinations of unemployment and inflation is formed by four countries - the Czechia, Germany, the Netherlands and Belgium. According to the translation coefficient (Table 1), Poland is very close to these countries ($\vartheta_{Pol,log} \approx \vartheta_{Pol,rate} \approx 1.11$). National “Phillips polygonal chains” of Romania, Greece, the Baltic countries, Spain and Slovakia ($\vartheta >$ 3.2) are the most distant from them (in ascending order). According to the authors, this means that without significant changes in their socio-economic policies (which would shift their national lines), these countries cannot significantly improve their international position.

The economies of countries located above the national optimum line can be characterised as "overheated" ones, and those located below it - as "overcooled" ones. In "overheated" economies, the specific weight of international unemployment losses is lower than the optimal level (54%), and in "overcooled" ones it is higher. According to both alternative methods, the economies of the Czechia, Poland, Hungary, Germany and Slovenia (in the order of increasing the specific weight of unemployment losses) are "overheated". For Malta and Bulgaria, inflation losses only slightly exceed unemployment losses (Table 4).

The group of the most "overcooled" economies included Cyprus, Denmark, Finland, France, Greece, Italy, Luxembourg, Portugal, Spain, Sweden and the European Union as a whole. In these economies, the share of international unemployment losses exceeded 90% (Table 4). Naturally, the question arises, what unites such different countries into one group? Moreover, the leading EU country, Germany, belongs to the opposite group of the most "overheated" economies, and the next largest economies, France, Italy and Spain, are "overcooled". Global macroeconomic shocks, such as the Covid-19 pandemic or energy price spikes, cannot explain such striking differences. The answer to this question lies in the peculiarities of the structures of these economies and the socio-economic policies of their governments. Most likely, the countries of the first group paid relatively more attention to combating unemployment than the countries of the second group. Because of this, the ratio of their international social losses shifted towards inflation. In the countries of the second group, the situation was the opposite. It is noteworthy that the European Union as a whole fell into the group of "overcooled" economies. This means that for it as a whole, it was inflation that was at the centre of regulatory policy. This feature of the Union is probably one of the reasons for its slower growth, in particular compared to the USA. It is also noteworthy that the three most "overheated" economies (the Czechia, Poland and Hungary) are not part of the euro zone. This gives them more space in changing their regulatory policies. The significant role of financing from EU funds is an additional feature of the Hungarian economy. After the full-scale Russian invasion of Ukraine, another important macroeconomic factor emerged. A large-scale migration of the population began from Ukraine. Its peculiarity was that the flow of migrants consisted mainly of people of working age. A significant proportion of migrants was engaged in business in their homeland and had enough funds to continue it abroad. Poland, Germany and the Czechia became the countries that accepted the largest number of Ukrainian migrants. This became a significant factor in "warming up" the economies of these countries. In the case of Germany and the Czechia, the increase in the production of military products that meet the conditions of the theatre of operations in Ukraine was an additional impetus.

The main information on the possibility of improving the international socio-economic position of individual countries is provided by national optimisation efficiency coefficients (Table 5). It is expected that for Germany and the Netherlands, which form the middle segment of the international efficiency frontier, the optimisation efficiency coefficient is equal to one - ($\epsilon_{Germ} = \epsilon_{Neth} = 1$). According to the proposed approach, this means that these two countries do not benefit from optimising their state on the international efficiency frontier. For Denmark, Luxembourg and Poland, the strategy of achieving the national optimum ($\mathcal{C}_i \longrightarrow \mathcal{O}_i$) and the strategy of direct transition to the international optimum ($C_i \longrightarrow O_{inter}$) turn out to be almost equivalent - ($\varepsilon_{log} \approx \varepsilon_{rate} \approx 1$). For Ireland, the strategy of direct transition to the international optimum would be preferable. Such a strategy would be even more profitable for three countries that are in middle segments of their own "Phillips polygonal chains" - Bulgaria, Malta and Slovenia.

From this point of view, it is interesting to compare two geographically neighboring countries - Bulgaria and Romania. For the Bulgarian economy, the effectiveness of the strategy of the gradual transition to the international optimum was only 39% of the effectiveness of the direct transition. For the Romanian economy, the situation was the opposite. For it, the effectiveness of the gradual transition was approximately 2.4 times higher than the effectiveness of the direct transition. A developed defense industry, which specialises in the production of Soviet-style weapons, is a special feature of the Bulgarian economy. Now these weapons have become an important factor in European aid to Ukraine. This may partly explain the differences in the specific share of unemployment losses. For Bulgaria, this indicator was about 48% of actual total losses, which is slightly less than the optimal ratio. For Romania, this indicator was much higher and was equal to 68% (Tables 4, 5).

It can be considered unexpected that efficiency coefficients of Greece and Spain turn out to be of the same order as the coefficients of the Czechia and Belgium - "borderline" countries of the international efficiency frontier ($1{,}37 < \epsilon_{log}(O_i) < 1{,}55$; $1{,}37 < \epsilon_{rate}(O_i) < 1{,}54$). This can be interpreted as a moderate (compared to other EU members) interest in moving along national Phillips DEA lines. It would be the most effective to optimise the states of Romanian and Slovak economies ($\epsilon(O_i) > 4$) and somewhat less effective - the states of the economies of Austria, Estonia, Latvia and Croatia ($\epsilon(O_i) > 3$).

The final comparison of countries' states according to two versions of the proposed model is carried out on the plane of initial "unemployment rate - inflation rate" indicators. The line of optimal states is the main tool of this analysis. The equation of this line, constructed according to the logarithmic version, has the following form:

$$\pi = 1/[(1 - u)\hat{}3.0325 \cdot e\hat{}0.0571] - 1 \qquad [31]$$

where π is the inflation rate, u is the unemployment rate, represented by decimal fractions. The line of optimal states corresponding to the version of relative increments is described by the equation

$$\pi = 3.0712u/(1 - u) - 0.0590 \qquad [32]$$

Outside the convex hull of actual states, the graph of the first function is located above (to the left of) the graph of the second one. However, within this hull, their graphs intersect twice, and in their middle part are very close to the linear function. As a result of this geometric feature, national optimal states calculated according to two alternative methods turn out to be the same for three countries - Ireland, Luxembourg and Poland (Table 3).

Imagining that countries with the winning strategy $C_i \longrightarrow O_i$ would move along linear trajectories in the direction of national optima, the following picture would form. The "overheated" economy of the Czechia in the process of optimisation would first pass through a state corresponding to the method of relative increments, and then, if the optimisation continues further, would reach a state with lower inflation, but with higher unemployment, which corresponds to the logarithmic method. The "overheated" economy of Hungary would follow a similar path with the difference that it would first reach the state calculated according to the logarithmic method and only then reach the point calculated according to the method of relative increments.

As for “overcooled” economies, most of them would first pass through an optimal state with lower inflation and higher unemployment, and then - an optimal state with lower unemployment but higher inflation. In this case, the countries with low inflation would first cross the line of optima according to the logarithmic method, and the countries with high inflation - according to the method of relative increments. However, six “overcooled” economies would be an exception to this pattern. For the economies of Cyprus, France, Italy, Portugal and Sweden, optimisation according to the method of relative increments would provide both lower unemployment and lower inflation. On the contrary, for the Spanish economy, a better result would be provided according to the logarithmic optimisation method.

6. Conclusions

The consistent adherence to a non-parametric approach in modeling and analysing the economic policy of the EU countries regarding the trade-off between unemployment and inflation is the main feature of the study. To implement this approach, a set of certain theoretical analysis tools, which were combined into a new version of the classical DEA model, was introduced for the first time.
The international social loss function, which makes it possible to assess the degree of deviation of the actual state of the country from the virtual “ideal” state $\mathcal{I}\big(f(u_{min}); f(\pi_{min})\big)$, is the first such tool. Like the popular quadratic function, this function is a set of elliptic curves, but unlike its analogue, it is constructed using a non-parametric method. To normalise the deviations of actual states of countries from the centre of elliptic curves, the point of the binomial mean of the states of effective countries $\mathcal{M}\left(\overline{f(u_{eff})}; \overline{f(\pi_{eff})}\right)$ was used for the first time. Due to this, inflation and unemployment targets obtained different weighting coefficients, determined in a nonparametric way.

The introduction of national Phillips DEA lines is the second new tool of analysis. Unlike traditional “inflation - unemployment” functions, the lines proposed by the authors were constructed using a non-parametric method. The point of tangency of the international social loss curve with the international DEA efficiency frontier (the lower left part of the convex hull of the states of countries on the plane of inflation and unemployment indicators) was interpreted as the international optimum $\mathcal{O}_{inter}$. National Phillips DEA lines were constructed by parallel transfer of the international efficiency frontier along the $\mathcal{I}\mathcal{O}_{inter}$ line. Due to this, the line of national optima was transformed into a straight line.

International optimisation costs are the third new tool of analysis. They are described by the same function as international social losses, but normalised deviations of actual states of countries from international and national optima are their arguments. The effectiveness of a certain optimisation strategy was estimated as the ratio of the social loss reduction to optimisation costs. Optimisation efficiency coefficients made it possible to find out which strategy is better for a certain country - a strategy of gradual improvement of its international position or a strategy of direct transition to the international optimum.

As shown in subsection 3.2, this version of the DEA model is not the only possible one. The quadratic form of the international social loss function and the method of normalising deviations from its centre are certain assumptions. Similarly, the idea of national Phillips DEA lines and the method of their construction are based on assumptions. However, in the authors' opinion, all other modifications of the model are either more arbitrary or create additional problems with justification or economic interpretation.

The proposed optimisation model was tested on EU data for 2023 in two alternative coordinate systems - on the plane of logarithms of indices and on the plane of relative increments. The study has revealed that there are a number of dependencies that are invariant with respect to the change in the coordinate system. According to both versions, the international frontier of efficient combinations of unemployment and inflation was formed (in order of location) by the Czechia (the country with the lowest unemployment rate), Germany, the Netherlands, and Belgium (the country with the lowest inflation rate). The difference in inflation rates in the Czechia and Belgium far exceeded the difference in their unemployment rates. As a result, social loss curves were extremely elongated along the inflation axis ($e > 0{,}95$), and the point of the international optimum was closer to the point of the Netherlands than to the point of Germany. In both coordinate systems, the share of optimal unemployment losses was almost equal ($\approx 54\%$). A comparison of optimisation efficiency coefficients for the strategies of step-by-step and direct transition to the O_{inter} state revealed a dependence that is important from a practical point of view. Namely, the second strategy would be better for countries closest to the national optimum line (Bulgaria, Malta and Slovenia), as well as for the country closest to the international Phillips DEA line (Ireland). For Denmark, Luxembourg and Poland, both strategies turned out to be almost equivalent. For countries that form the middle segment of the international efficiency frontier (Germany and the Netherlands), the gain from reducing social losses is completely neutralised by the costs of changing their state.

It is worth emphasising, however, that the results obtained are only a “snapshot” for the period under study. The increase in its duration will raise the question of a certain averaging of the initial data and will make it possible to stabilise the conclusions of the model. In this case, the averaging of the initial data can be carried out by various methods. According to the first method, the mean state of each country is constructed on the basis of its actual states for all the studied years. According to the second method, only those states of the country that form its own efficiency frontier will be the basis for averaging. Other, more subtle methods are also possible. Similarly, the coordinates of the “ideal” and “terrible” states are averaged in the case of using elliptic or hyperbolic functions of international social losses. A larger value of the studied period makes it possible to move from the classic DEA model to its parameterised versions. For this, normalised deviations from extreme states will first be squared, and then rise to the η power, determined parametrically. Binomial weighting coefficients used in constructing the mean state point $\mathcal{M}\left(\overline{x_{eff}};\overline{y_{eff}}\right)$ can be supplemented with terms that also take into account other factors. In particular, the duration of the “leader” country's stay on the international efficiency frontier may be such a factor.

In the extended version of the proposed model, it will be possible to analyse short- and long-term DEA optima separately. The expansion of the chronological framework of the study will make it possible to conduct a thought experiment, namely, to try to find out how the actual state of countries would change if they began to take into account international and national DEA optima. The expansion of the statistical base will make it possible to investigate which countries were approaching the optimal DEA states and which ones were moving away. The comparative analysis of

DEA optima with optimal states on parametric Phillips curves may become a separate problem of future research.

However, the greater capabilities of the extended versions of the model have their own “price”. Their construction faces certain problems - how many years to choose for research and how to take into account the aging of data over time.

The version of the model presented in this paper implicitly assumes that a country's movement towards the international optimum requires significantly greater changes than the movement towards the national optimum.

The first strategy is based on a certain trade-off between moving along the parametric (traditional) Phillips curve and moving along its DEA analogue. Depending on how its targeted state and the state of the national DEA optimum are mutually located, the country can additionally stimulate its domestic demand, or, conversely, somewhat restrain it. In this case, measures to stimulate domestic demand do not necessarily have to be monetary or fiscal in nature. As the modern experience of Canada and many European countries has shown, a spontaneous movement to increase the consumption of domestic goods and services can create a significant impetus for the national economy.

As for the strategy of moving towards the international DEA optimum, its implementation requires serious structural reforms, the greater the further the country is from it. Although EU countries have different demographic structures and different labour legislation, in general, Europe stands out against the background of other regions of the world with a larger share of elderly people and greater labour market rigidities. In the current conditions, European pensioners value price stability more, while workers value job guarantees, social assistance and growth in their real wages. However, the continued aging of the population, the escalation of geopolitical rivalry between the United States and China, and the aggressive policies of totalitarian countries - all this increases the risks to European economic and social stability. The solution may consist in a number of reforms - a certain unification of the EU labour legislation, partial deregulation of the labour market, greater requirements for recipients of social assistance, and greater control over migration flows from non-European countries. These reforms should be organically combined with a policy of stimulating innovative business and European cooperation in the most advanced areas of technological progress. The acceleration of European development will also be facilitated by the continuation of the EU's active foreign economic policy in other regions of the world. This is especially true for the most "Europeanised" countries, such as Canada and Latin American countries.

All these measures and possible risks of their implementation require separate analysis and may become the subject of future research using both parametric and non-parametric methods.

The proposed model can be used in various areas.

First, the theoretical substantiation of socio-economic policies of national governments is the most obvious area of its application. National social losses and the optimisation efficiency coefficient calculated on its basis can be used as an argument for changing the policies of the government, central bank and relevant national legislation. The model can be used as an additional tool for differentiated targeting of the main macroeconomic indicators, which is carried out by governing institutions of the European Union, and in particular by the European Central Bank.

Second, the mathematical form of the proposed model can be easily transferred to other socio- and financial-economic indicators, for which it is advisable to determine the international DEA efficiency frontier. The increase in the dimensionality of the model will make it possible to calculate generalised translation and optimisation efficiency coefficients. Such generalised coefficients can be taken into account when constructing various international indices and rankings of countries. The indices and rankings constructed in this way can become one of the foundations for the policies of international organisations and transnational corporations. On the other hand, they themselves can act as an object for applying the proposed model.

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