REVISTA DE ESTUDIOS E INVESTIGACIÓN EN PSICOLOGÍA Y EDUCACIÓN
eISSN: 2386-7418, 2017, Vol. Extr., No. 06
DOI: https://doi.org/10.17979/reipe.2017.0.06.2307

Teacher knowledge and the teaching of statistics using a graphing calculator

Conhecimento profissional e ensino de estatística com recurso à calculadora gráfica

Helena Rocha
Faculdade de Ciências e Tecnologia – Universidade NOVA de Lisboa

Abstract

Teaching statistics is often based on an approach focused on teaching theoretical aspects, disconnected from practical relevance and from interpretation of results, and where the use of technology lies behind its potential. In this context, it is important to analyze how the teachers' knowledge is characterized and to identify aspects of this knowledge that mark the professional practice. The conclusions reached emphasize the impact of content knowledge and its influence on knowledge of content and teaching. Knowledge of curriculum is also relevant, as well as the way how it seems to prevent the development of other types of knowledge.
Keywords: Teachers' knowledge, statistics, technology.

Resumo

As abordagens adotadas pelos professores assentam numa abordagem à estatística focada no ensino de aspetos teóricos, desligados de relevância prática, da interpretação de resultados e onde a utilização da tecnologia fica aquém do que poderia. Importa pois analisar como se carateriza o conhecimento do professor e identificar aspetos desse conhecimento que marcam a prática profissional. As conclusões alcançadas enfatizam o impacto do conhecimento do conteúdo e a sua influência sobre o conhecimento do conteúdo e do ensino. É ainda relevante o conhecimento do currículo e a forma como este obsta ao desenvolvimento dos demais conhecimentos.
Palavras chave: Conhecimento profissional, estatística, tecnologia.

Introduction

Statistical knowledge has assumed an increasing importance in society. Every day, we face several situations requiring interpretation based on some kind of statistical knowledge. This evolution led the school to integrate this theme into the curricula of Mathematics. But this theme differs from other mathematical content and, besides that, statistics is a theme in which many teachers have not had enough training. Several studies have pointed to some limitations in teachers' practice, suggesting that their origin may be based on aspects of the teacher's professional knowledge. Technology is also becoming an indispensable and integral part of today's society, but once again it is something relatively new and whose integration into the learning process remains below its potential. In these circumstances, it is important to analyze how the teachers' knowledge to teach statistics in a context of technology integration is characterized and to identify aspects of this knowledge with a strong impact on the professional practice. Only in this way it will be possible for teachers' training programs (initial or continuous) to become a relevant element for the professional development of teachers. As so, this study specifically intends to understand the teachers' knowledge and conceptions regarding statistics; and to characterize the teachers' professional practice, especially with regard to the type of tasks proposed to the students and to the role reserved for the graphing calculator.

The teacher and the teaching of statistics

There are several characterizations of teacher professional knowledge developed over the years but, whether we base ourselves on the work of Shulman (1986) or on some other more recent work such as the one of Hill and Ball (2009), all characterizations emphasize the importance of different types of teachers' knowledge. In this study we will adopt the MKT - Mathematical Knowledge for Teaching model (Hill & Ball, 2009) and we will focus on some of the domains of knowledge defined there (see Figure 1).

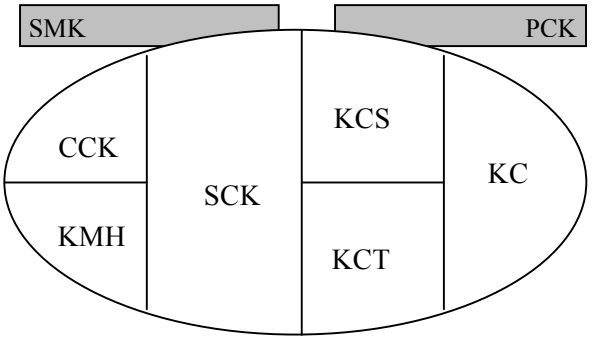


Figura 1. MKT – Mathematical Knowledge for Teaching (Hill & Ball, 2009)

Hill and Ball (2009) draw on Shulman's work and consider two major areas: SMK - Subject Matter Knowledge and PCK - Pedagogical Content Knowledge. In the scope of the first, they consider CCK - Common Content Knowledge, SCK - Specialized Content Knowledge and KMH - Knowledge at the Mathematical Horizon. And in the context of the

Correspondence: Helena Rocha, hcr@fct.unl.pt
Selección y *peer-review* bajo responsabilidad del Comité Organizador del XIV Congreso Internacional Galego-Portugués de Psicopedagogía

second, they consider KCS - Knowledge of Content and Students, KCT - Knowledge of Content and Teaching and KC - Knowledge of Curriculum. This is a widely known model, so we refrain from presenting here the characterization of each of the above domains.

Studies conducted specifically in the field of statistics teaching point to the low level of knowledge held by the teachers. And, according to Nicholson, Ridgway and McCusker (2006), there is even a consensus about the need to improve the initial and continuous training of teachers in this area. Indeed, as Kader and Perry (2006) report, statistics is a relatively new topic for many teachers who have not had the opportunity to develop knowledge about the principles and concepts behind the data analysis they are now being asked to teach. These teachers cannot see statistics in the curriculum in a global and coherent way, nor can they understand the differences between mathematics and statistics.

According to several authors, statistics is a mathematical science but it is not mathematics. Statistical reasoning is different from the mathematical one for several reasons, being the main one the question of the uniqueness of the solution (Gattuso & Pannone, 2002). In statistics the problems usually begin with a question, but ask more for an opinion based on assumptions and data analysis, than a single and indisputable answer as it happens in Mathematics. The focus is not on an appreciation from the type right or wrong of the conclusions and inferences presented by students. On the contrary, what is appreciated is the quality of the reasoning, the adequacy of the methods used and the evidence used. In addition, while in mathematics teaching, the use of real contexts is recommended, because it is considered to contribute to the understanding of concepts, in statistics it is considered essential that students perceive data as numbers in context used to address a specific question (Burgess, 2006).

Being able to analyze and understand statistical information has become, in today's society, fundamental to the exercise of citizenship (Galmacci & Milito, 2002). As such, the school started to aim the education of statistically literate citizens, able to interpret the data transmitted by the media and to question the conclusions proclaimed by them (Kader & Perry, 2006).

Studies have concluded that the approaches adopted by teachers and textbooks are based on a statistical approach focused on the teaching of theoretical aspects, disconnected from their practical relevance and the interpretation of results (Galmacci & Milito, 2002). As some authors have pointed out, statistics is taught in the mathematical style, placing more emphasis on formulas and calculations than on the interpretation of results (Gattuso & Pannone, 2002). And if access to technology allows shifting the focus of teaching from the execution of calculations and graphs with paper and pencil to reasoning and statistical thinking, there are other reasons that hinder this change. On the one hand, conceptual ignorance of elementary notions (such as the measures of central tendency), documented in studies involving teachers or future teachers (see, for example, Batanero (2000)), constitutes a strong obstacle to the development of a practice that goes beyond performing calculations. On the other hand, it is important to take into account the teacher's beliefs and conceptions regarding teaching, since they will interfere with the options taken by the teacher, in favor of an approach with a greater emphasis on calculation or with a greater emphasis on improving the communication and argumentation capacity.

Methodology

The nature of the issues involved in this study led to the adoption of a qualitative methodology. A case study was conducted involving a young Mathematics teacher, Sandra, and accompanying her work with a 10th grade class.

Several instruments were used to collect data. Four 90-minute classes were observed during the 3rd term, when the teacher was teaching statistics. During the school year, the material developed by the teacher (work sheets, tests, etc.) was collected and at the end of the year one student's notebook was also collected. In addition, two semi-structured interviews were conducted with the teacher. The first interview took place in the 3rd term, after the observation of the first class, and it assumed a special form. It started with the presentation to the teacher of a proposal of work for a class of statistics and with a first moment in which it was suggested to the teacher to try it. The interview in itself began later, with a request for comments on the task, and it was centered around the views expressed by the teacher. The second interview took place shortly after the end of classes and focused on various aspects relevant to the study. Throughout the school year, there were also several informal talks.

The teacher and the statistics

When speaking about statistics, Sandra refers to a pleasant subject, mentioning that it gives her pleasure to teach it. She highlights the great proximity between this theme and reality and points this as a factor that establishes a difference between statistics and other mathematical themes. She then justifies her empathy for the theme with its application to reality and with the ease with which we come across it on a daily basis, on a newspaper or on television. But when trying to achieve a better understanding over the reasons that turn statistics into an enjoyable subject to her, it seems she is referring more to the reasons why she likes to teach it than to the reasons that justify her personal opinion over the subject:

> I like it because it's so... it's all around us, isn't it? And if we read a newspaper or a magazine we can easily find graphs and things like that... and when I get to class and I talk or show a graph, they already know... they've seen it before... and they feel that it has something to do with them, with their life... and they want to see it, they are more interested... And besides that, statistics it's also easier, isn't it?... They realize that... and this also makes them like and... none of this happens with other content...

When I try to make her think only on statistics, and not on its teaching, she ends up saying that it is not an area she particularly likes because she finds it a little bit heavy and full of complicated formulas:

> I think it's a lot like… just those awful formulas and then calculations and more calculations. (...) I don't like it so much.

However, after presenting her idea about statistics, she clearly distinguishes between her opinion about statistics in general and her opinion about secondary statistics. And she emphasizes her appreciation for secondary statistics, emphasizing its closeness to reality.

Regarding her knowledge of statistics, Sandra says that this is a subject in which she feels quite comfortable, referring to the high level of demand of the university she attended. Still, during the first interview, when she was offered a task that required not the calculation of the mean of a set of data, but the understanding of the notion, she revealed some difficulties.

The task proposed took the form of a game in the graphing calculator. The game consisted in the presentation of a set of five numbers and the value corresponding to its mean, and the player was asked to add another element to the data so that the mean becomes equal to another value also given. Once the requested element was introduced, the game presented the new mean and, if it was not close enough to the requested mean (it was not required to reach the exact value, and the acceptable approximation could be defined by the player), the game will request once again for the introduction of another element so that the new set of data has the desired mean. The more values you would need to enter, the lower the score obtained at the end of the game.

Sandra did not show any difficulty in understanding what was asked, but the observation of the data set, its mean, and the new mean sought, did not seem to give her any clue about the new value to be considered. After experiencing the game for some time, even introducing values that changed the mean of the new set of data away from the desired value, she ended up picking a sheet of paper and writing an equation ((sum of the five data+x)/6 = new mean) that she solved to find the requested value.

Professional practice in statistics' teaching and the graphing calculator

In one of the classes devoted to statistics, the work began with five sheets moving around the class. These sheets had a grid, vertically containing the number of all students in the class and horizontally four aspects such as height (in cm), eye color, number of people with whom they live and preferred discipline. Each student had to fill in the appropriate space the data that concerned him. After the data collection was completed, the sheets, which were all different, were distributed to the groups of students that were supposed to handle the information. In a final phase of the class, each group would present to their classmates the characterization of the class regarding the variables they had studied.

During the class the teacher assumed a somewhat directive position, progressively defining aspects that the initial work proposal left to the students' discretion. Thus, she ended up stipulating that for each of the four variables the students had to present a graph and a result (mean, mode or median) and that, as far as possible, both the results and the graphs should be of varying types. This action had the consequence of reducing the task to a simple exercise. A discussion about the suitability of choosing, for example, to calculate the mean or the median, was something that did not took place.

In another lesson, the task proposed to the students requires them to prepare tables of frequency, to draw graphs and to calculated certain measures of central tendency or dispersion.

During the study of the topic, the students were never confronted with situations where they had to discuss the criteria inherent to choose a sample, process data or presenting results. For example, they have never had a chance to see how a change in the range of classes can change the appearance of an histogram or how certain inaccuracies, eventually used deliberately by advertising, can promote distorted information. All the work accomplished had a procedural basis and assumed a strong emphasis on calculations.

Sandra justifies her options based on the time available and the need to comply with the program:

> I wish I could have done a little statistical study with them... (...) but it was not possible. These things always take a lot of time and I didn't have many lessons. I needed to slow down in the functions, I could not go forward without ensure they realize certain things they will need later, isn't it? Here, in statistics, that is not quite the same...

However, some of the options that she has assumed are difficult to understand in the light of this argument of lack of time. And the main one is related to the use of the calculator. The students had to learn to use the machine, that required time, but then they were asked to perform all the calculations by themselves, not being allowed to determine the mean or the standard deviation of a set of data using the calculator. Confronted with this incongruity, Sandra ends up showing some discomfort:

> Yes, I know. But it's a bit like this, if I allow them to do all the calculations on the machine, then what is done by them? The machine does everything, doesn't it? (...) Here it is not like the functions in which you have to know how to use, know how to choose the window and all these things and for this you have to know and use Mathematics...

In fact, the analysis of the student's notebook regarding the classes focused on the study of functions shows that in this case the calculator has been widely used to draw graphs and to search for relations between them and the expressions of the associated functions. It is possible to notice that this technology was used as a support for problem solving, for the implementations of small explorations and even for a modeling task. The tasks proposed by Sandra in the scope of the study of

statistics and functions have very different characteristics, as does the role reserved for technology.

In addition, Sandra's comments on the game presented to her, which involved the concept of mean, illustrate well that the domain of concepts in the field of statistics is not valued by her:

> I do not think this is... a very interesting task. The students might enjoy it, (...) but I think the focus should be on learning and here... this game will not help them learn how to calculate the mean of a set of data, isn't it?

Discussion of the results

Kader and Perry (2006) refer to differences between mathematics and statistics and Gattuso and Pannone (2002) argue that statistics is not mathematics and reinforce this difference by emphasizing that statistics is reasoning and justifying an opinion based on findings and assumptions and never just calculation. A statistical perspective that clearly contradicts that of Sandra, who values calculations and neglects the interpretation of results and the mastery of concepts, but which, according to the latter authors, is common among mathematics teachers.

Doerr and Zangor (2000) associate the teachers' conceptions regarding Mathematics to the type of use that is made of the calculator, considering that a greater valuation of the concepts or the calculation correspond, respectively, to a view of the calculator as a source of learning or as an obstacle. Sandra attributes a great importance to calculation in the teaching of statistics. This option corresponds to a conception of the use of the calculator as an obstacle to learning. However, this does not seem to illustrate Sandra's conceptions about teaching. In fact, the tasks she proposed during the teaching of functions and the use of the calculator associated with them point to a conception of the use of the calculator as a source for learning.

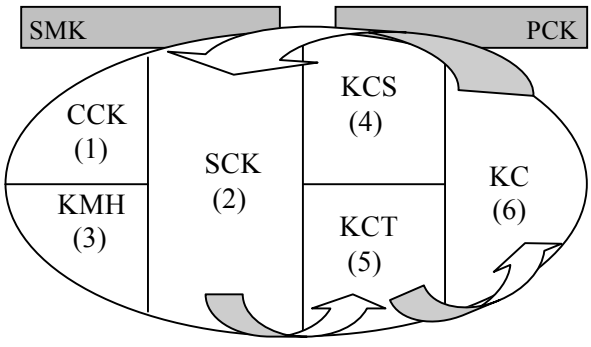
Batanero (2000) considers that the valuation of calculation in the teaching of statistics is not due to a conviction of the teacher. In his view, such a situation simply reflects a narrow mastery of concepts, which prevents the teacher from taking advantage of technology and adopting a practice that goes beyond calculations. In Sandra's case, it seems to be true that she does not have a profound domain of the concept of mean, and it is admissible that this may also be true for other statistical concepts. Even so, it is not necessarily this the reason for her preference for an emphasis on calculation. The truth is that her work in the field of functions suggests that she is not a teacher who strongly values calculations in mathematics teaching, which in a certain way can support the idea defended by Batanero, but does not exclude others factors that can also be present.

Another factor that some authors refer to is the manual, a recognized influence on teachers' practice. And according to several studies mentioned by Galmacci and Milito (2002), they rarely emphasize the understanding of statistical concepts and the interpretation of situations. But the manual alone is not enough to exclude the reason pointed out by Batanero, since it is the lower teachers' knowledge that makes them lean more on the manual and adopt in a less reflected way the perspective adopted by it. Sandra's knowledge of statistics will thus be a relevant and influential element in her practice.

However, the one that seems to be the determining factor, is a devaluation of the statistic in relation to the other subjects. In other words, it is the teacher's knowledge of the curriculum that leads her to value certain subjects in detriment of others. Sandra makes reference to it saying that functions are more important because gaps at this level may have implications for the future of her students. On the contrary, she does not show this concern in relation to statistics, where she consciously reduced the number of lessons. Statistics seem to be viewed as a theme that students like and where they do not face difficulties, allied to the idea of a lack of consequences for the future. It may thus make sense that the investment in the conceptual development of students under this theme is lower.

Conclusion

The conclusions reached show a clear appreciation on the part of the teacher of calculation, in detriment of the interpretation of results and the mastery of the concepts in what relates to the teaching of statistics. This is one aspect that can be promoted by the features of the adopted manual, but which seems to be strongly influenced by other reasons. The teacher's knowledge of the subject seems to be centered in the execution of calculations, not including a full mastery of the concepts. This prevents her from recognizing the learning potential of concept-focused work proposals and, as a result, ends up constituting a limitation of her knowledge in teaching and learning. The valuation of the calculation leads her to feel a dilemma about the use of technology that immediately performs all calculations. She ends up by not being able to take advantage of technology to the learning of the students



(an evidence of her reduced knowledge of how to teach statistics with the calculator).

Figure 2. Synthesis of the main idea on each domain of knowledge

(1) Statistics is calculations and formulas
(2) Statistics is calculation with real data
(3) Statistics has no impact on students' future

(4) Statistics is something familiar and easy
(5) Appropriate tasks for the teaching of statistics are focused on calculation. We cannot allow the use of calculator.
(6) Statistics is a minor issue

The devaluation attributed to the theme, when compared to other mathematical themes, justifies the disinvestment she makes in its teaching. It thus reflects the knowledge and conceptions that she has of the curriculum and the mathematical horizon of the students, where statistics is a theme with less impact on the future mathematical learning.

Overall, it is the knowledge of the content held by the teacher, where the valuation of calculations is dominant, which prevents her from developing her knowledge of content and teaching and also makes evident some disarticulation with regard to the integration of the calculator. In turn, the knowledge she holds of the curriculum prevents her from recognizing the relevance of teaching and learning statistics and consequently realizing the importance of developing her knowledge to teach and change her practices (see Figure 2).

The existence of a kind of cycle of influences between different domains of knowledge (see Figure 3) suggests that in the case of this teacher the professional development should focus on the development of statistical knowledge in relation to KCT with a direct focus on tasks.

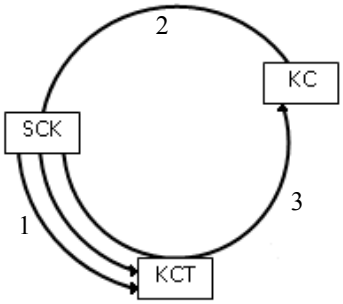


Figure 3. Cycle of influence between different knowledge.
(1) Focus on calculation determines the type of proposed tasks
(2) Statistics' devaluation reinforces focus on the calculation
(3) Valuation of the calculation reinforces devaluation of the theme

The conclusions of this study emphasize the importance of teacher-training programs focused on statistics and the different types of knowledge that a teacher mobilizes in his/her practice (and the identified fragilities associated with it), as a way of enhancing professional development.

References

Batanero, Carmen (2000). Dificuldades de los estudiantes en los conceptos estadísticos elementares: el caso de las medidas de posición central. In C. Loureiro, F. Oliveira & L. Brunheira (Eds.), *Ensino e aprendizagem da estatística* (pp. 31-48). Lisboa: SPE, APM, DE e DEIO da FCUL.

Burgess, T. (2006). A framework for examining teacher knowledge as used in action while teaching statistics. In A. Rossman & B. Chance (Eds.), *Proceedings of the International Conference on the Teaching of Statistics - ICOTS 7*. Salvador: IASE.

Doerr, H. & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics*, 41, 143-163.

Galmacci, G. & Milito, A. (2002). The effects of some teaching techniques on learning statistics. In *Proceedings of the International Conference on the Teaching of Statistics - ICOTS 6*. South Africa: IASE.

Gattuso, L. & Pannone, M. (2002). Teacher's training in a statistics teaching experiment. In *Proceedings of the International Conference on the Teaching of Statistics - ICOTS 6*. South Africa: IASE.

Hill, H., & Ball, D. (2009). The curious – and crucial – case of Mathematical Knowledge for Teaching. *Phi Delta Kappan*, 91(2), 68-71.

Kader, G. & Perry, M. (2006). A framework for teaching statistics within the K-12 mathematics curriculum. In A. Rossman & B. Chance (Eds.), *Proceedings of the International Conference on the Teaching of Statistics - ICOTS 7*. Salvador: IASE.

Nicholson, J., Ridgway, J. & McCusker, S. (2006). Reasoning with evidence – developing curriculum. In A. Rossman & B. Chance (Eds.), *Proceedings of the International Conference on the Teaching of Statistics - ICOTS 7*. Salvador: IASE.

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.